Rational Buyer Meets Rational Seller: Reserves Market Equilibria under Alternative Auction Designs

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Abstract

We examine efficiency properties and incentive compatibility of alternative auction formats that an electricity network system operator may use for the procurement of ancillary services required for real-time operations. We model the procurement auction as a hierarchichal multiproduct auction and study several designs such as a uniform price auction minimizing social cost, a uniform price auction minimizing the system operator's cost and a pay as bid auction. We take into account that rational bidders will respond to any market design so as to maximize their expected benefit from participating in that market. Under the assumptions of our model, we show that the uniform price auction minimizing social cost is the only one that guarantees productive efficiency. We also find that expected revenue (payment in our case) equivalence between pay as bid and uniform price auction does not extend to the hierarchical products case and the ranking of these auction is ambigious and depends on the data. For the procurement auction minimizing the system operator's cost we show that misrepresentation of capability may result in capacity shortages if there are capacity constraints. For the case where only higher capability resources are constrained this will result in random price spikes decreasing in frequency with the price cap (this is the amount paid to capacity in demand states with shortages). When lower type resources are capacity constrained as well, price spikes will be seen for both type of resources. Artificial shortages result in reduced reliability in real-time operations.

1 Introduction

In many restructured electricity markets in the US and around the world, a system operator is responsible for the real time control of the transmission system. In California, as in the Pennsylvania-New Jersey-Maryland Interconnection (PJM), New York Power Pool (NYPP), New England Power Pool (NEPP) and the proposed Electricity Reliability Council of Texas (ERCOT) market, this role is performed by an Independent System Operator (ISO) that is responsible for real time load balancing, congestion management and provision of ancillary services. These services include voltage support, black start capability, automatic generation control (AGC) and reserves with varying levels of response time. The precise definition of ancillary services varies across different restructured systems and so do current market designs for competitive procurement and provision of such services. In many markets such as California, NEPP, NYPP and ERCOT, separate ancillary service products have been defined which are procured by the ISO through an auction on behalf of market participants so as to meet National Electricity Reliability Council (NERC) reliability standards. In this paper we examine alternative auction formats that the ISO may use for the procurement of ancillary services.

Proper product definition and design of ancillary services markets are the primary determinants of efficiency and liquidity in these markets. This in turn influences system reliability. This lesson was learned in California through the poor performance and low liquidity of its ancillary services market that necessitated price caps and other forms of direct intervention by the ISO (California ISO Market Surveillance Committee 1999). In New England, the New England ISO market report for June 1999 (also see Cramton and Lien 2000) points out various flaws such as price reversals, low demand elasticity, and sequential market clearing as factors that hinder the smooth operation of reserves markets there. These flaws resulted in numerous administrative interventions overriding market prices and eventually in the temporary suspension of the ancillary services market operations in New England. Likewise in New York, NYPP filed on March 27, 2000 an emergency request to the Federal Energy Regulatory Council (FERC) to suspend market based pricing of 10-Minutes Reserves due to market failure that was attributed to the excercise of market power. Market reforms aimed at repairing the ancillary services markets are under way in all these ISOs.

A key feature of these markets is the downward substitutability among the various reserve types (i.e., faster response reserves can be regarded as high quality resources and can substitute for slower response reserves). Hence, price reversals in these markets create perverse incentives for generators and may lead to misrepresentation of capability and bids. The remedies proposed or adopted by the different systems vary with emphasis on the specific problematic issues in each system. In New York, the main focus of the reform is on reducing market power of the ancillary services suppliers by recognizing the congestion problems between east and west that limit competition in the 10-minutes reserve market. In New England, the focus of the reform is on introducing price elasticity in the reserve markets and modifying the dispatch protocol in order to neutralize the gaming that plagued those markets. In California, the primary objectives of the reform have been to improve liquidity and efficiency in the ancillary services markets and reduce procurement cost. Key elements of the California reform have been to move from a sequential to a simultaneous least-procurement-cost auction for the various services and to allow the ISO to exploit the substitutability among the different reserve types. Specifically, the California ISO (CAISO) needs to procure four types of reserves: Regulation (RG) for AGC [1], Spinning reserves (SP) that are synchronized and available within 10 minutes, Non-Spinning reserves (NS) that are not synchronized but can be made available within 10 minutes and Replacement reserves (RS) that can be made available within 60 minutes. These products are, for practical purposes, hierarchically substitutable. Up-regulation resources can also provide SP, NS and RS. Likewise, spinning reserves can provide NS and RS whereas non-spinning reserves can provide RS. In the initial market design, the auction for the four reserve products was conducted sequentially from the highest to the lowest quality. Generators were allowed to rebid uncommitted resources in any round at new prices in subsequent rounds. Thus, they could exploit thin markets in the lower quality reserves. The CAISO reforms for ancillary services procurement attempt to stabilize prices by permitting generators to submit only a single bid specifying reserve type, quantity, and a capacity-energy price pair. The ISO can use any of the procured resources to meet demand for any of the reserve products that a resource can provide (this is often referred to as cascading bids). So, for instance, the ISO may procure spinning reserves and use them to meet the need for replacement reserves.

Within the framework of a simultaneous multiproduct auction with possible cascading of bids, there are several alternatives for organizing the auction that define how winning bids are selected and how payoffs are determined. These alternatives have diverse efficiency and distributional implications, which have been recognized and debated by the ISO board. The scheme that has been adopted in California employs a "rational buyer" bid selection criterion that minimizes total procurement cost and pays to winning bids of each reserve type (as declared by the bidder) a uniform market clearing price set by the highest accepted bid of that type (or a pre-specified price cap if there is a shortage for that reserve type). In New York, on the other hand, the bid selection is based on minimizing social cost as reflected

[1] AGC is composed of two services, upward regulation and downward regulation. Upward regulation is reserve capacity on generators that can be used by the ISO in real time. Downward regulation is already running generation that can be backed down by the ISO in real time. We will only consider upward regulation in our analysis as only this type is substitutable for the other reserve types.

by the bids and market clearing prices for each reserve type are set to the marginal cost of providing that reserve (this equals the highest accepted bid for all reserves of equal or lower quality). Yet another auction format is the pay as bid auction which has been adopted for the balancing market in the UK under the New Electricity Trading Arrangements (NETA). In this auction, the ISO minimizes revealed social cost and pays each selected bidder their bid.

The main objective of this paper is to analyze the implications of these design alternatives with respect to efficiency and incentive compatibility. We examine the implications of these designs taking into account that rational bidders will respond to any market design so as to maximize their expected benefit from participating in that market. We also explore revenue (payment in our case) equivalence between the auction formats to determine if the move away from more efficient and equitable market designs produces the desired results, such as reducing procurement cost, in the case of California.

The next section provides a review of the relevant literature. We review, both, the auction theory literature for results on multiproduct, multipart and multidimensional auctions as well as studies on auctions related to electricity. In Section 3 we examine the design options in greater detail, state our assumptions and present data used in the numerical examples. In Section 4 and 5 we study the three auction formats for four data sets with different incentive compatibility properties. In Section 6 we examine the case where there are capacity constraints. Finally, we offer some concluding remarks in Section 7.

2 Review of Relevant Literature

The bidding literature addressing multiproduct auctions where each bidder bids for a single unit dates back to Vickery (1962). Vickery analyzed uniform price and discriminatory auctions (also called pay as bid auctions in procurement settings) and proved that his famous revenue equivalence result for single unit auctions (Vickery 1961) extends to this multiunit case. Vickery assumed that one auctioneer is selling a given number of indivisible units to a given number of risk-neutral bidders, and symmetric and independent uncertainty of reservation values among the bidders. Since then the literature has focused on relaxing these assumptions and analyzing the case where bidders have multiunit demands. A comprehensive survey of this literature is provided by Klemperer (1999) (see McAfee and McMillan 1987 for a literature review on single unit auctions). Another important aspect studied in the auction theory literature is the optimality of various auction formats given the objective of maximizing the sellers expected revenues. Myerson (1981) and Riley and Samuelson (1981) show that first and second price auctions for single units are both optimal. Harris and Raviv (1981) extend this result to the case of multiple units where each bidder bids for a single unit and establish optimality of discriminatory and uniform price auctions for this case. To the best of our knowledge there have been no studies of simultaneous multiproduct auctions where the products are hierarchically substitutable.

The literature on multiunit auctions addresses cases where bidders can bid for multiple units and the case where a single divisible unit is being auctioned and bidders can bid for a share of the item. Engelbrecht-Wiggans (1988) established expected revenue equivalence between multiunit uniform price and discriminatory auctions under the assumption that bidders can bid for a subset of the items at the same price. Maskin and Riley (1989) show, however, that standard auction procedures are no longer optimal when bidders can bid for multiple units. Instead the optimal mechanism is a nonlinear pricing scheme. Wilson (1979) analyzed a divisible good auction where bidders bid for a share of the item on sale. He showed that there can be some collusive looking equilibria in a uniform price auction where bidders shade their demands and prices are low. Back and Zender (1993) and Wang and Zender (1995) also study this type of auction and find that revenue equivalence results do not generally hold in this case. They also find that the ranking of discriminatory and uniform price auctions may be ambiguous. Ausubel and Cramton (1998) (and references therein) study divisible good auctions with a focus on their efficiency properties. They find that the uniform price auction is susceptible to bid shading under a number of assumptions which has an adverse impact on efficiency. Bidders shade their bid at higher quantities because there is a positive probability that it is the price setting bid in which event they earn greater profits on their inframarginal units. Such bidding behavior has been studied in an empirical setting for the England and Wales electricity system by Wolfram (1998). The pay as bid auction does not suffer from this problem as each unit is bid on it own merit and this auction format is more likely to attain an efficient outcome. The advantages of pay as bid auctions in avoiding collusive behavior are discussed by Klemperer (2000). Ausubel and Cramton also study the ranking of the two auction formats and find that the ranking is ambiguous and depends on the data.

Literature focusing on bidding in the context of electricity markets initially addressed PURPA auctions for Independent Power Producer (IPP) contracts. Bids in these auctions consisted of two parts, a capacity payment due upfront, and an energy bid for electricity production during the course of the contract. Kahn, Rothkopf, Eto and Nataf (1990) examine efficiency aspects of the acquisition process but do not treat cases where bids are curtailable. Stoft and Kahn (1991) examine questions concerning "bias" in scoring of curtailable bids but do not have an explicit treatment of the trade-off between fixed and variable price components. Bushnell and Oren (1993 and 1994) address the question of whether bidders in the Biennial Resource Planning Update (BRPU) auction mandated by the California Public Utility Commission (CPUC) for procurement of IPP capacity by the utilities, will be induced to reveal their "true" fixed and marginal cost of generation. Such information was essential for efficient procurement and dispatch of procured resources. Their analysis predicts that the proposed CPUC scoring rule is likely to induce understatement of marginal generation cost (which indeed happened). The outcome of that auction, which was eventually voided by FERC due to "gaming behavior" by the bidders are discussed by Gribik (1995). Bushnell and Oren (1994) also propose a Vickery auction type scheme for which true revelation of marginal costs is a dominant strategy for all bidders.

Green and Newberry (1992) analyze bidding behavior in the context of the UK electricity. Using the supply function equilibria technique developed by Klemperer and Meyer (1989) they show that bids above marginal cost can be supported in equilibrium. Von der Fehr and Harbord (1993) is the first paper we are aware of that analyzed a multiunit auction for electricity in the context of the UK power pool. They show that pure strategy supply function equilibria derived in Green and Newberry do not generalize to the case where bidders have to bid steps functions. However, pricing above marginal cost is seen in many equilibria that they derive. More recently, Green and McDaniel (1999a and b) and Green (1999) analyze bidding behavior for the new market rules proposed under the Reform of Electricity Trading Arrangements (RETA) in the UK electricity market. They model choices faced by a small generator who can sell into the forward market or the balancing market run by the National Grid Company (NGC). For the balancing market alone they show expected revenue equivalence between a uniform price auction and a pay as bid auction under demand uncertainty and common knowledge of the merit order. They also show that if there are two independent draws for demand in the balancing mechanism and generators have the choice of adjusting their bid after the first realization the revenue equivalence result would continue to hold.

The earlier UK market design, like the later market designs adopted in PJM, NYPP and New England, preserved the central unit commitment aspect of the vertically integrated utilities by means of a multipart auction in which bidders provide the necessary inputs for central unit commitment optimization. Unfortunately, multipart auctions are not well understood and with few exceptions, that we will mention below, have limited theoretical foundation that would enable a practical incentive compatible design. Johnson, Oren and Svoboda (1997) illustrated some inherent difficulties arising from central unit commitment in a competitive environment with dispersed generation ownership. The difficulties are due to indeterminacies of the optimal unit commitment solution and basic incompatibility between competitive behavior and truth telling due to integer affects and non-convexities. Wilson (1998) and Elmaghraby and Oren (1999) have proposed alternative designs using single-part auctions that enable bidders to internalize non-convexities due to fixed cost. Hobbs et al. (1998) studied an incentive compatible multipart electricity auction based on the

Vickery-Clarke-Groves mechanism. Unfortunately, as noted by the authors, this approach has limited practical value due to revenue deficiency and the discriminatory nature of the payment scheme.

Reserves markets can be viewed as special types of multipart auctions where generators compete for the provision of reserves by submitting two-part bids consisting of capacity and energy prices. Chao and Wilson (1999) develop an incentive compatible design for the special case of a single reserve type. In their proposed scheme generators submit separate capacity and energy bids. The energy bids are used in case it is necessary to call the reserves to supply energy and all dispatched energy is remunerated uniformly at a price set by the highest dispatched energy bid. They prove that under this setup a scoring rule that ranks reserves bids based only on their capacity component and pays all accepted bids a capacity price set to the highest accepted capacity bid is incentive compatible, i.e., bidders will be induced to reveal their true marginal energy cost as well as their capacity cost (this may be an opportunity cost such as forgone profit from sale of energy).

The problems associated with the California ancillary services market that led to the "rational buyer" reform are documented in the Annual report of the California ISO market surveillance committee (1999) and discussed by Laura Brien (1999). Some of the deliberations concerning the ISO ancillary services market reform have been documented in unpublished memoranda. The solution to the rational buyer auction given bids can be cast as an mathematical program. Oren (2000) and Chicco and Gross (undated) and references therein consider solution strategies to the rational buyer problem. Oren proposes a dynamic programming algorithm that makes use of the special structure of the problem. An analysis of alternative market designs for ancillary services is contained in the Griffes (1999) report to the California Power Exchange that is considering establishing its own internal ancillary services market for the purpose of self-provision. However, none of these discussions explicitly address bidders' strategic response to the market design.

3 Design Options and Model Details

As discussed earlier, an important aspect of ancillary services is their hierarchical nature that allows substitution of a high quality reserve for a lower quality one. Both social efficiency and rational procurement behavior dictate that such substitution should be allowed. In a perfectly competitive market such substitution would occur naturally in a sequential auction (from high to low quality) since bidders would rebid their rejected bids in the subsequent auctions for which their resources are eligible. In principle, assuming bidders bid and rebid their true cost, such a sequential auction would lead to socially efficient procurement. In the absence of market power, uniform market clearing prices in each auction will indeed induce bidders to bid their true cost. Unfortunately, a sequential auction with independent uniform market clearing prices in each round may result in price reversal, i.e., the market clearing price for a high quality resource (e.g. regulation) may be lower than that for a lower quality resource (e.g. spinning reserves). Indeed, such price reversals have been observed in California and New England ancillary services markets. Price reversals pose serious incentive compatibility problems since even price taking generators, anticipating such reversal, may be induced to understate their capability and wait for a later round of the sequential auction that is expected to fetch a higher market clearing price. With market power, the situation is exacerbated as bidders who were not selected in early rounds may raise their bids in subsequent rounds when they perceive potential scarcity of bids. Furthermore, losing bids in the earlier rounds may end up being paid more than the winning bids if selected for service. Clearly, this price reversal creates perverse incentives for withholding bids or raising prices in the earlier rounds. This is true even for resources that have no market power and cannot affect market clearing prices but simply try to take advantage of the intermarket arbitrage opportunity by selling their product in the auction that will fetch the best price. In reality, bidders in the California ancillary services market also exploited market power due to low liquidity and raised their bids in the subsequent auction rounds (see Oren 2000 for an example of price reversal in sequential auctions).

The proposed reform to the California ancillary services markets replaces the sequential auction with a simultaneous auction where each resource submits a single bid specifying reserve type, capacity bid and energy price if called. The ISO is allowed to substitute demand for a lower quality reserve with higher quality, and thus, use higher quality reserves to meet demand for lower quality reserves. Similar schemes are used in NYPP and were recommended for New England by Cramton and Lien.

Within the above simultaneous auction framework there are several degrees of freedom in the market design:

1. The objective function of the bid selection protocol
   (a) Minimum social cost
   (b) Minimum procurement cost
2. Settlement rule
   (a) Pay uniform price based on bid type (demand substitution)
   (b) Pay uniform price based on usage (product substitution)
   (c) Marginal cost pricing
   (d) Pay as bid

3. Pricing of the products to buyers
   (a) Set product price to highest accepted bid of that type
   (b) Set product price to highest price paid to meet product demand
   (c) Marginal value pricing

Much of the deliberations in the California ISO focused on the choice of objective function for implementing the rational buyer approach. The implication of that choice depends on the settlement rule. Paying resources based on usage (product substitution) is a natural choice in sequential auctions where rejected resources in one auction can rebid or are carried over to the next. In such a case, the payments are based on the clearing price of the auction in which a resource wins. Under such a settlement rule, however, if a rejected bid in an early auction is accepted in a later one (assuming no bid change) it follows that the clearing price in the later auction (for lower quality reserves) will be higher than the auction that cleared earlier in which the bid was rejected. Avoiding such situations is one of the main arguments in favor of a simultaneous auction. However, a simultaneous auction in which resources are paid based on usage is de facto equivalent to a sequential auction. Therefore, we will not consider that option any further.

The remaining settlement rules are uniform payment based on bid type, uniform payment based on marginal cost, and pay as bid. To date, a pay as bid approach has been adopted only in the UK under the New Electricity Trading Arrangements (NETA). Under the uniform pricing options the two possible bid selection objectives with demand substitution can lead to different dispatch results. In particular, the procurement cost minimization option adopted by the CAISO may result in a socially inefficient dispatch and in price reversal as will be shown below. Oren (2000) shows that even if bids are selected so as to minimize social cost price reversals could still occur if procurement prices are not set to the marginal cost of providing a reserve.

In our study, we analyze three simultaneous auction formats for a two product ancillary services market. The two products are modeled as hierarchically substitutable, e.g. regulation and spinning reserves. Regulation capacity can be used as spinning reserves but not vice-versa. In a competitive single-product market, a uniform pricing rule guarantees productive efficiency. Therefore, we first examine a uniform price auction minimizing social cost (MSC auction). In the MSC auction, bidders are paid the marginal value of serving the corresponding reserve type (i.e. the highest quality reserve that a bid can provide). We then consider a uniform price auction minimizing the ISO's cost (Rational Buyer or RB auction). The RB auction is designed to minimize procurement cost given bids and it sacrifices some efficiency to achieve this objective. Finally, we study a pay as bid auction (PaB auction) in which the ISO minimizes revealed social cost and pays each selected bidder their bid. Our primary areas of focus in this analysis are the efficiency properties of the various auction formats and the incentive compatibility in terms of truthful bidding of capability (type) and cost.

3.1 Model Assumptions and Data

In all our models we consider infinitesimal generators that have no market power. The cost function parameters are common knowledge and therefore the generators know their position in the particular merit order to which they belong. The details of the selection protocol is also common knowledge and this implies that generators will have rational expectations about the behavior of other generators and will bid accordingly.

We examine two types of data. First, we examine a case where there are no price reversals in the RB auction. This will be the case when there are no capacity constraints (in the relevant range) and type 1 costs dominate type 2 costs. Then, we consider data for which there will be price reversals in the RB auction. Again, with no capacity constraints this is the case when type 2 costs dominate type 1 costs (in the uncertain part of demand). Price reversals can also occur under capacity constraints for type 1 service and we will also examine the problem under capacity constraints.

We assume that type 1 and type 2 demands are uncorrelated and uniformly distributed on $[r_{10}, r_{10}+r_1]$ and $[r_{20}, r_{20}+r_2]$, respectively[2]. We also assume that the merit orders have costs that are increasing linearly in the quantity supplied given by:

$$\begin{aligned} c_1(q_1) &= \beta_1 + \gamma_1 q_1 \\ c_2(q_2) &= \beta_2 + \gamma_2 q_2 \end{aligned} \tag{1}$$

We can invert this function and write:

$$\begin{aligned} q_1(c_1) &= -\frac{\beta_1}{\gamma_1} + \frac{c_1}{\gamma_1} \\ q_2(c_2) &= -\frac{\beta_2}{\gamma_2} + \frac{c_2}{\gamma_2} \end{aligned} \tag{2}$$

In the PaB and MSC auctions the lower supports can be ignored as selection is based on marginal costs only. For the RB auction the quantity procured for each type becomes important as revenues are a function of quantity as well as price. Supports are explicitly accounted for in this case and we assume all generators that will called on with certainty

[2] We assume uncorrelated uniform demand uncertainty in both demands which is 8-10 percent of average demand for that type of service. We also assume that average type 1 demand is forty percent of average type 2 demand which is consistent with historical data on the ratio of regulation to other type of reserves.

have cost equal to the intercepts of the cost functions. Table 5 in the Appendix shows the data used in the numerical examples.

4 No Price Reversals - Data Set 1

We first study the effiency properties of the MSC auction. Under the assumptions of our model, we show that bidders have a dominant strategy to bid their true capability and cost. As the objective function used in the selection procedure is minimizing social cost, this guarantees productive efficiency. Incentive compatiblity, with respect to capability, comes from the fact that there are no price reversals in the MSC auction. Also, as small generators cannot affect price in a uniform price auction, bidders maximize their profit by bidding their true cost. We then contrast the RB auction against this format and show how the efficient choice is distorted under the RB auction format to reduce total procurement cost. In this section, we examine the case where there are no price reversals in the RB auction. We first analyze the RB auction under the constraint that generators bid their true types. We then relax the assumption of true type disclosure and show that the true type equilibria will continue to hold. This implies that the RB auction is incentive compatible for Data Set 1. Finally, we study a PaB auction. We find that the PaB auction is also incentive compatible with respect to capability. However, bidders bid more than their cost as they trade-off lower probability of selection against increased profit from bidding above cost. Consider two equal cost generators, one in each merit order. Given a bid level, the type 1 generator's probability of selection will be greater than or equal to the type 2 generator's probability. This is due to the downward substitutability of the reserve types. Therefore, the type 1 generator will have greater expected profit. This also implies that the bid premia for the two generators will not be the same. Therefore, the PaB auction will not achieve productive efficiency as the bid-based efficient choice of generators will not correspond to the cost-based efficient choice.

We compare expected payments from the three auction formats and find that standard revenue equivalence results between PaB and MSC auctions cannot be extended to the hierarchical products case. The weak form of the expected revenue equivalence theorem states that if the probability of selection of each generator is the same in two auctions and (in our case) the generator with the highest cost from among those that can be potentially selected makes zero profit then expected payments in the two auctions are the same. As the PaB auction does not lead to efficient outcomes the probability of selection for generators is different as compared to the MSC auction and in general the expected revenue equivalence does not go through. We cite some numerical results that may shed some light on the ranking of these auction formats.

4.1 Uniform Price Auction Minimizing Social Cost - MSC Auction

In a uniform price auction for procuring a single service type, all generators bid their cost in equilibrium. This is because a small generator cannot affect the price it is paid by altering its bid. If it increases its bid above cost, this reduces the probability of selection and the generator is not selected for some demand realizations in which it would have made a profit. Also, there is no equilibrium with any generator bidding below cost. Although, this does increase the probability of selection, it does not change the prices that the generator receives when it is selected for demand realizations greater than q_1, its place in the merit order. When demand is below this level the generator makes a loss for every demand realization it is selected.

This logic extends to the case of 2 service types. In Data Set 1, $c_1(r_1) > c_2(r_2)$. This means that some type 1 generators are in a single demand world. Here, the above logic applies directly. For the remaining type 1 generators, if one bids above cost (given what everyone else is bidding), this reduces its probability of selection for type 1 service and leaves out some profitable demand realizations. Also, this generator will spill over to the type 2 auction at a higher bid and thus have a lower probability of selection for type 2 service than if it were to have bid cost. The prices it receives cannot be changed in any case. Thus, bidding cost results in an equilibrium for a simultaneous hierarchical uniform price auction that minimizes social cost.

We begin by examining the MSC auction for n hierarchichally substitutable products, i = 1, 2, $\ldots n$ (see Oren 2000 for a four product example). Each infinitesimal bidder declares its type and bid. The ISO then constructs supply functions, $p_i(q_i)$ which give the marginal cost for supplying quantity q_i (net of lower supports). We denote the cumulative capacity acquired of type i or higher by $Q_i = \sum_{j=1}^{i} q_j$. We denote the cumulative demand for type i or higher as $D_i = \sum_{j=1}^{i} d_j$ where d_j is demand for type j.

Given supply functions and demand realizations, the ISO solves a mathematical program minimizing Cost-of-Production (CoP):

$$\begin{aligned} \min_{q_1, q_2 \cdots q_n \geq 0} CoP &= \sum_{i=1}^{n} \int_0^{q_i} p_i(\phi_i)\,\mathrm{d}\phi_i \\ s.t. \quad & Q_i \geq D_i \; for \; i = 1, 2, \ldots n \end{aligned} \tag{3}$$

and determines optimal procurement quantities, q_i^* and prices, p_i, for $i = 1, 2, \ldots n$. A simple greedy algorithm will solve the problem. Procure quantity D_1 for meeting type 1 demand. Now pool the remaining type 1 supply with the type 2 supply available and select the least-cost bids to meet type 2 demand. Repeating this procedure for all types will guarantee an optimal solution. We state the no price reversal result as Proposition 1.

Proposition 1 *There are no price reversals in the MSC auction, i.e.* $p_1 \geq p_2 \geq \ldots \geq p_n$.

Proof. See Appendix.

Given that bidders will reveal their true cost in the auction, minimizing revealed cost of production is equivalent to minimizing social cost of production and the MSC auction will result in productive efficiency. We qualify our results here by saying that they are applicable only for the case of infinitesimal generators. As discussed in Section 2 this may not continue to hold when there are generators of positive size. It is likely that in this situation generators will raise bids on higher quantity units so that if these units set the price it increases their profits on inframarginal units. This will have an adverse impact on efficiency.

We are now in a position to calculate expected payments that the ISO makes for our two demand case. As the cost functions are continuous, prices are decided based on the marginal generator selected of each type, irrespective of what service it provides[3]. Prices will therefore depend on both demand levels. This leads to Proposition 2.

Proposition 2 *Expected payments in a uniform price auction are given by:*

$$\begin{aligned} ER^{MSC}(s_1) &= \int_{s_1}^{r_1} \int_{0}^{\min\{d_2^*, r_2\}} (\beta_1 + \gamma_1 d_1)\, \mathrm{d}F_2(d_2)\mathrm{d}F_1(d_1) \\ &= + \int_{s_1}^{r_1} \int_{\min\{d_2^*, r_2\}}^{r_2} (\beta_3(d_1) + \gamma_3 d_2)\, \mathrm{d}F_2(d_2)\mathrm{d}F_1(d_1) \\ &\quad + \int_{0}^{s_1} \int_{\min\{s_1 - d_1 + q_2(\beta_1 + \gamma_1 s_1), r_2\}}^{r_2} (\beta_3(d_1) + \gamma_3 d_2)\, \mathrm{d}F_2(d_2)\mathrm{d}F_1(d_1) \end{aligned}$$

$$\begin{aligned} ER^{MSC}(s_2) &= \int_{\min\{d_1^*, r_1\}}^{r_1} \int_{s_2}^{\min\{q_2(\beta_1 + \gamma_1 d_1), r_2\}} (\beta_2 + \gamma_2 d_2)\, \mathrm{d}F_2(d_2)\mathrm{d}F_1(d_1) \\ &\quad + \int_{\min\{d_1^*, r_1\}}^{r_1} \int_{\min\{q_2(\beta_1 + \gamma_1 d_1), r_2\}}^{r_2} (\beta_3(d_1) + \gamma_3 d_2)\, \mathrm{d}F_2(d_2)\mathrm{d}F_1(d_1) \\ &\quad + \int_{0}^{\min\{d_1^*, r_1\}} \int_{\min\{d_1^* - d_1 + s_2, r_2\}}^{r_2} (\beta_3(d_1) + \gamma_3 d_2)\, \mathrm{d}F_2(d_2)\mathrm{d}F_1(d_1) \end{aligned} \tag{4}$$

[3] For no price reversals to occur, Oren (2000) shows that when there are discontinuities in the cost function generators will have to be paid at the marginal cost of supplying a resource type, which may be different than the marginal cost of the last generator selected.

where d_1 and d_2 and demands for type 1 and type 2 service, respectively. $d_2^* = q_2(\beta_1 + \gamma_1 d_1)$, $d_1^* = q_1(\beta_2 + \gamma_2 s_2)$ $\beta_3(d_1)$ and γ_3 are intercept and slope, respectively, of type 2 supply when demand for type 1 service is d_1.

Proof. See Appendix.

Figure 1 shows expected profit for equal cost generators in an MSC auction (this can be calculated using profit margins instead of prices in the above integrals). Observe that type 1 generators have higher expected profit than equal cost type 2 generators and this auction format is incentive compatible for Data Set 1.

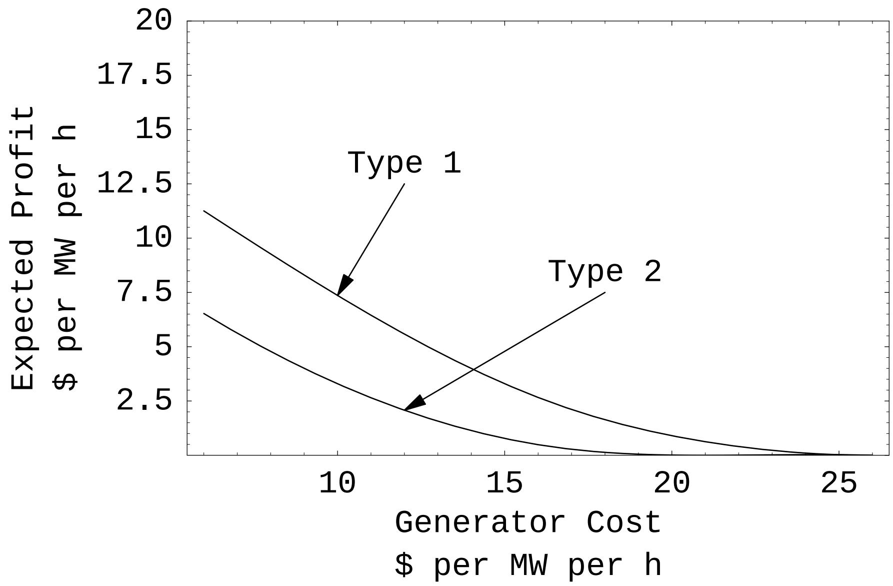

Figure 1: Expected Profit in an MSC Auction (Data Set 1).

4.2 Uniform Price Auction Minimizing ISO's Cost - Rational Buyer

The Rational Buyer protocol is a simultaneous uniform price auction that minimizes the ISO's procurement cost. As there are concerns about incentive compatibiilty in this auction we first analyze the auction under the constraint that generators reveal their true type. We call this type of RB auction a constrained RB (cRB) auction. We now show that bidding cost in a cRB auction results in an equilibrium. First, we describe the problem facing the ISO. Given supply functions and demand realizations, the ISO solves a minimum Cost-to-Procure (CtP) problem:

$$\min_{q_1,q_2\cdots q_n\geq 0} CtP = \sum_{i=1}^{n} P_i(q_i)$$
$$s.t. \quad Q_i \geq D_i \, for \, i = 1, 2, \ldots n \tag{5}$$

where $P_i(q_i) = q_i p_i(q_i)$, the total payment to resource type i as a function of the acquired capacity and other terms as defined above. The objective of this problem is nonconvex for general supply functions and we use the dynamic programming algorithm described in Oren (2000) to solve this problem. The dynamic program can be formulated as:

$$\min_{D_n\geq Q_{n-1}\geq D_{n-1}} \begin{bmatrix} P_n(D_n - Q_{n-1}) \\ + \min_{Q_{n-1}\geq Q_{n-2}\geq D_{n-2}} \begin{bmatrix} P_{n-1}(D_{n-1} - Q_{n-2}) \\ \ldots + \min_{Q_2\geq Q_1\geq D_1} \begin{bmatrix} P_2(D_2 - Q_2) \\ +P_1(Q_1) \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{6}$$

This is an n stage dynamic program. The stages in the DP formulation represent the resource types in the hierarchy, while the states are the cumulative amounts of resources acquired at each stage. The resource quantities need to be discretized using an appropriate increment depending on the precision required. The solution of the DP involves one forward pass and one backward pass. In the forward pass we start with the innermost minimization and compute the cost of acquiring any feasible quantity of type 1 resource between the demand for type 1 capacity and the combined demand of all n types. Next, we compute the least cost feasible mix of type 1 capacity and type 2 capacity for any total amount of the two in the range between the combined demand for type 1 and type 2 and the demand for all services, subject to the constraint that we at least satisfy type 1 demand with type 1 capacity. This is done for all n resources. In the backward pass we start with the total amount of the n ancillary services and trace the least cost path from which we can extract the optimal quantity of each resource type. Prices can be determined from the supply functions (see Oren 2000 for an example with 4 types).

For the data in this section we can find an analytical solution to the above mathematical program (5). The Karush-Kuhn-Tucker (KKT) conditions for this program can be solved to get:

Case 1: 1^{st} constraint inactive; 2^{nd} constraint active.

$$\begin{aligned} q_1(d_1, d_2) &= \frac{\beta_2 - \beta_1}{2(\gamma_1 + \gamma_2)} - \frac{\gamma_1 r_{10} - \gamma_2 r_{20}}{2(\gamma_1 + \gamma_2)} + \frac{\gamma_2(d_1 + d_2)}{\gamma_1 + \gamma_2} \\ q_2(d_1, d_2) &= -\frac{\beta_2 - \beta_1}{2(\gamma_1 + \gamma_2)} + \frac{\gamma_1 r_{10} - \gamma_2 r_{20}}{2(\gamma_1 + \gamma_2)} + \frac{\gamma_1(d_1 + d_2)}{\gamma_1 + \gamma_2} \end{aligned} \tag{7}$$

Case 2: Both constraints active.

$$\begin{aligned} q_1(d_1, d_2) &= d_1 \\ q_2(d_1, d_2) &= d_2 \end{aligned} \tag{8}$$

We can now state Proposition 3.

Proposition 3 *Bidding cost results in an equilibrium of the cRB auction.*

Proof. See Appendix.

One criticism of the Rational Buyer model is that under certain cost and demand conditions it can lead to price reversals (i.e. lower type generators are paid more than equal cost higher type ones), thus making the auction violate incentive compatibility constraints. One should remember, however, that when demand is uncertain, even if instances of price reversals do occur, the auction will violate incentive compatibility only if expected profit for a type 1 generator under the constrained auction are lower than for a type 2 generator with the same cost. We characterize the range of data where no price reversals occur in Proposition 4.

Proposition 4 *For no price reversals to occur in a RB auction we must have:*

$$\gamma_2 r_{20} - \gamma_1 r_{10} \geq 0$$

Proof. See Appendix.

Data set 1 has $\gamma_2 r_{20} - \gamma_1 r_{10} = 0$[4]. Therefore, the RB auction for this data should be incentive compatible. In order to check this, we calculate expected payments for the cRB auction in Proposition 5 (expected profit can be calculated by replacing the price term by a profit margin term in all the integrals).

Proposition 5 *Expected payments in a cRB auction are:*

$$\begin{aligned} ER^{cRB}(s_1) = & \int_0^{d_2^*}\int_{s_1}^{r_1} p_1^{case2}\,\mathrm{d}F_1(d_1)\mathrm{d}F_2(d_2) + \int_{d_2^*}^{d_2^{***}}\int_{s_1}^{r_1} p_1^{case2}\,\mathrm{d}F_1(d_1)\mathrm{d}F_2(d_2) \\ & + \int_{d_2^{***}}^{r_2}\int_{\min\{d_1^*(d_2), r_1\}}^{r_1} p_1^{case2}\,\mathrm{d}F_1(d_1)\mathrm{d}F_2(d_2) + \int_0^{d_1^{**}}\int_{d_2^{**}(d_1)}^{r_2} p_1^{case1}\,\mathrm{d}F_2(d_2)\mathrm{d}F_1(d_1) \\ & + \int_{d_1^{**}}^{r_1}\int_{d_2^{**}(d_1)}^{r_2} p_1^{case1}\,\mathrm{d}F_2(d_2)\mathrm{d}F_1(d_1) \end{aligned}$$

[4] We choose parameters in Data Set 1 also keeping in mind that the analytical solution needs to be used here.

$$ER^{cRB}(s_2) = \int_{s_2}^{r_2}\int_{\min\{d_1^*(d_2),r_1\}}^{r_1} p_2^{case2}\,\mathrm{d}F_1(d_1)\mathrm{d}F_2(d_2) + \int_0^{d_1^{**}}\int_{d_2^{**}(d_1)}^{r_2} p_2^{case1}\,\mathrm{d}F_2(d_2)\mathrm{d}F_1(d_1)$$
$$+ \int_{d_1^{**}}^{r_1}\int_{\min\{d_2^*(d_1),r_2\}}^{r_2} p_2^{case1}\,\mathrm{d}F_2(d_2)\mathrm{d}F_1(d_1) \tag{9}$$

where p_i^{casej} is the price for service type i, for Case j described above.
$d_2^* = \frac{\beta_1-\beta_2}{2\gamma_2}$; $d_2^{***} = \frac{\beta_1-\beta_2}{2\gamma_2} + \frac{\gamma 1}{\gamma_2}s_1$; $d_1^*(d_2) = \frac{\beta_2-\beta_1}{2\gamma_1} + \frac{\gamma 2}{\gamma_1}d_2$; $d_2^{**}(d_1) = \frac{\beta_1-\beta_2}{2\gamma_2} + \frac{(\gamma 1+\gamma_2)}{\gamma_2}s_1 - d_1$; and d_1^{**} is be derived by solving $d_2^*(d_1) = d_2^{**}(d_1)$.

Proof. See Appendix.

To examine incentive compatibility we need to see how expected profit of equal cost generators in the two merit orders compare. Figure 2 shows expected profit of equal cost generators. Observe that type 1 generators get higher expected profit than equal cost type 2 generators and thus even if the constraint of true type bidding were removed this would not change bidding behavior in this auction. We note here that as we expect $r_{20}/r_{10} > 1$, we would need $\gamma_2/\gamma_1 < 1$. We state the incentive compatibility result as Proposition 6.

Proposition 6 *Under no capacity constraints, the RB auction satisfies incentive compatibility when type 1 costs dominate type 2 costs and* $\gamma_1 r_{10} - \gamma_2 r_{20} = 0$.

Proof. See Appendix.

4.3 Pay as bid Auction

In a PaB auction the ISO minimizes revealed social cost and pays each selected bidder their bid. We begin by writing down expected profit of a generator with q_1 MW above it in the merit order (i.e. with lower costs). If this generator bids b (with $s_1(b)$ MW above it in the merit order), its expected profit can be expressed as:

$$\Pi_1(b) = (b - c_1(q_1))\Big((1 - F_1(s_1(b))) + \int_0^{s_1(b)} (1 - F_2(s_1(b) + s_2(b) - s))\,\mathrm{d}F_1(s)\Big) \tag{10}$$

where $c_1(q_1)$ is cost of a type 1 generator at q_1 in the merit order, $s_1(b)$ and $s_2(b)$ are the amounts of type 1 and type 2 bidders bidding less than b, respectively, and, $F_1(s)$ and $F_2(s)$

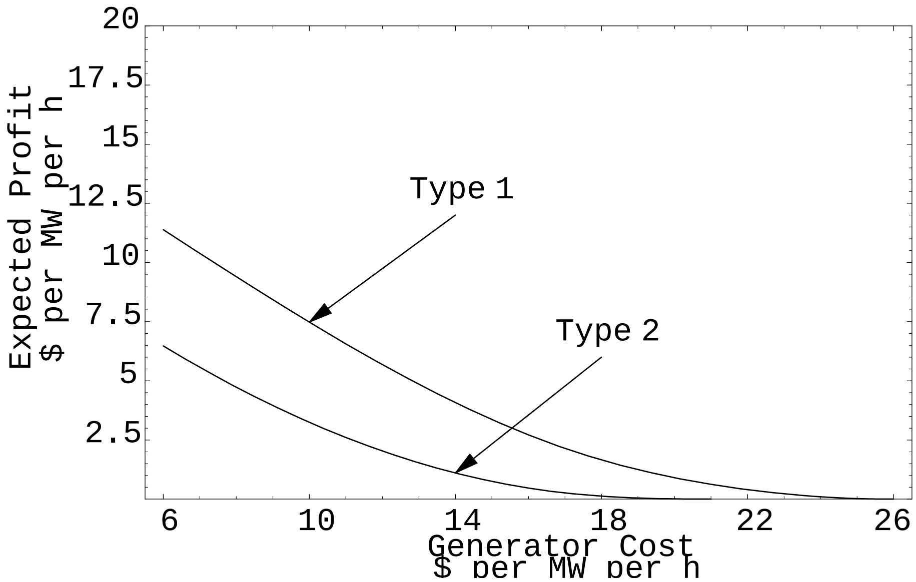

Figure 2: Expected Profit in a cRB Auction (Data Set 1).

are the cumulative distributions of type 1 and type 2 demand, respectively.

The first term is the profit margin that a generator with cost $c_1(q_1)$ will receive when selected for service at bid b. The first term in brackets after the profit margin term is the probability that this generator is selected for type 1 service, i.e. that demand is greater than $s_1(b)$. The second term is the probability that the generator is selected for type 2 service, i.e. that type one demand is lower than $s_1(b)$, but type two demand is greater than what is available at bid b, $s_1(b) - s + s_2(b)$, where, s is type 1 demand.

Expected profit of a type 2 generator with q_2 MW above it in the merit order and bidding b is:

$$\begin{aligned} \Pi_2(b) = {} & (b - c_2(q_2)) \Big(\int_0^{s_1(b)} (1 - F_2 (s_1(b) + s_2(b) - s))\, \mathrm{d}F_1(s) \\ & + (1 - F_1(s_1(b)))\ (1 - F_2(s_2(b))) \Big) \end{aligned} \tag{11}$$

where $c_2(q_2)$ is cost of a type 2 generator at q_2 in the merit order.

The probability of selection of a type 2 generator depends on the demand for type 1 service. When this demand is low, more low cost type 1 generators will be moved over to the type 2 auction. The first term in brackets is the probability that a type 2 generator is selected for service given that type 1 demand was such that some generators with bids lower than b were moved over to the type 2 auction. The second term is the probability that a type 2 generator is selected for service given that type 1 demand was such that only generators with bids above b (and thus, not competing with our type 2 generator) were moved over to the type 2 auction (this term has the above form due to our assumption that the demands are independent).

Profit maximizing generators face a trade off between increased profit from bidding above costs and lower probability that they will be selected for service if they raise their bids. The first order necessary conditions for an optimal bid, b, are:

$$
\begin{aligned}
\frac{\mathrm{d}\Pi_1(b)}{db} &= 0 = (1 - F_1(s_1(b))) + \int_0^{s_1(b)} (1 - F_2\left(s_1(b) + s_2(b) - s\right)) \,\mathrm{d}F_1(s) \\
&\quad + (b - c_1(q_1)) \Big(- f_1(s_1(b))s_1'(b) + (1 - F_2(s_2(b)))\, f_1(s_1(b))s_1'(b) \\
&\quad + \int_0^{s_1(b)} (-f_2\left(s_1(b) + s_2(b) - s\right)) \,\mathrm{d}F_1(s) \left(s_1'(b) + s_2'(b)\right) \Big) \\
\frac{\mathrm{d}\Pi_2(b)}{db} &= 0 = \int_0^{s_1(b)} (1 - F_2\left(s_1(b) + s_2(b) - s\right)) \,\mathrm{d}F_1(s) \\
&\quad + (1 - F_1(s_1(b)))\ (1 - F_2(s_2(b))) + (b - c_2(q_2)) \Big((1 - F_2(s_2)\, f_1(s_1(b))s_1'(b) \\
&\quad - \int_0^{s_1(b)} f_2\left(s_1(b) + s_2(b) - s\right) \mathrm{d}F_1(s) \left(s_1'(b) + s_1'(b)\right) \\
&\quad + (1 - F_1(s_1(b))\ \left(-f_2(s_2(b))\, s_2'(b)\right) + (1 - F_2(s_2(b)))\ \left(-f_1(s_1(b))\, s_1'(b)\right) \Big)
\end{aligned} \tag{12}
$$

If the distribution functions of the uncertain demands and the cost functions are known in closed form, one can arrive at two differential equations in the inverse bid functions, s_1 and s_2, that can be integrated by applying appropriate initial conditions. As the cumulative distribution function (CDF) of a uniform random variable is non-differentiable at the supports, we solve the above differential equation in several parts. Thus, there are 4 distinct regions of bids over which we integrate the ODEs.

Part 1: As $c_1(r_1) > c_2(r_2)$ there will be some type 1 generators that will never be selected for type 2 service (those with cost greater than $c_2(r_2)$). These generators will see demand as uniformly distributed in the range $[q_1(c_2(r_2)), r_1]$ conditional on when it is in that range. In the profit function of the generator the bid premium will be multiplied with only the first term in brackets. This is a single demand bidding problem (see Green 1999). The marginal generator at r_1 will bid cost as it cannot sustain a bid above cost in equilibrium.[5]

[5] If the marginal bidder bids above cost, the best response of inframarginal bidders is to increase their bids as this increases their profit without decreasing the probability of being selected. However, the best response

Proposition 7 (Green '99) *Bidders with bids above $c_2(r_2)$ will bid:*

$$b(s_1) = \frac{1}{1 - F_1(s_1)} \int_{s_1}^{r_1} (\beta_1 + \gamma_1 s)\, \mathrm{d}F_1(s) \tag{13}$$

Proof: See Appendix.

Part 2: At $b = c_2(r_2)$ the total capacity available is greater than r_2 (as $s_2(c_2(r_2)) = r_2$ by similar reasoning as above). Thus, we can calculate a b^* such that:

$$s_1(b^*) + s_2(b^*) = r_2 \tag{14}$$

The profit functions for $b^* \leq b \leq c_2(r_2)$ are given by:

$$\begin{aligned}
\Pi_1(b) &= (b - c_1(q_1)) \Big((1 - F_1(s_1(b))) \\
&\quad + \int_{\max\{s_1(b)+s_2(b)-r_2,0\}}^{s_1(b)} \int_{\min\{s_1(b)+s_2(b)-d_1,r_2\}}^{r_2} \mathrm{d}F_1(d_1)\mathrm{d}F_2(d_2) \Big) \\
\Pi_2(b) &= (b - c_2(q_2)) \Big(\int_{\max\{s_1(b)+s_2(b)-r_2,0\}}^{s_1(b)} \int_{\min\{s_1(b)+s_2(b)-d_1,r_2\}}^{r_2} \mathrm{d}F_1(d_1)\mathrm{d}F_2(d_2) \\
&\quad + (1 - F_1(s_1(b)))\, (1 - F_2(s_2(b))) \Big)
\end{aligned} \tag{15}$$

For some demand realizations total capacity available for type 2 service at bid b exceeds r_2 (after deducting type 1 demand). In these states, the probability that a generator who bids b will be selected for type 2 service is 0. Therefore, the amount of capacity available for type 2 service will be less than r_2 for $d_1 > s_1(b) + s_2(b) - r_2$. One can derive the above first order conditions using this range of (d_1, d_2). Initial conditions for part 2 are:

$$\begin{aligned}
s_1(c_2(r_2)) &= s_1^* \\
s_2(c_2(r_2)) &= r_2
\end{aligned} \tag{16}$$

where s_1^* is calculated from the bid function in part 1 at $b = c_2(r_2)$.

---

of the marginal generator to this response is to under-cut an inframarginal bidder. Now the marginal bidder has a positive probability of being selected and will make a strictly positive profit. One can see that this cannot be sustained and in equilibrium the marginal bidder bids cost.

Part 3: For $b < b^*$, there is always a strictly positive probability that a type 1 generator who is in a type 2 auction gets selected for service. The profit function in this case will have 0 as the lower limit on the range of d_1 and the lower limit on d_2 is $s_1(b) + s_2(b) - d_1$. Initial conditions for part 3 are:

$$\begin{aligned} s_1(b^*) &= s_1^{**} \\ s_2(b^*) &= s_2^{**} \end{aligned} \tag{17}$$

where s_1^{**} and s_2^{**} are calculated from the bid functions in part 2 at $b = b^*$.

This set of ODEs is solved till $s_1(b) = 0$. This is the minimum bid that will be submitted by a type 1 generator, b^{1min}.

Part 4: At bids below b^{1min} the profit function for type 2 generators will be independent of type 1 demand and will resemble Part 1 of the analysis. Proceeding in a similar fashion we can derive the bid function for type 2 generators in this region (see Figure 3). The initial condition for this set of ODEs is:

$$s_2(b^{1min}) = s_2^{***} \tag{18}$$

where s_2^{***} is calculated from the bid functions in part 3 at $b = b^{1min}$.

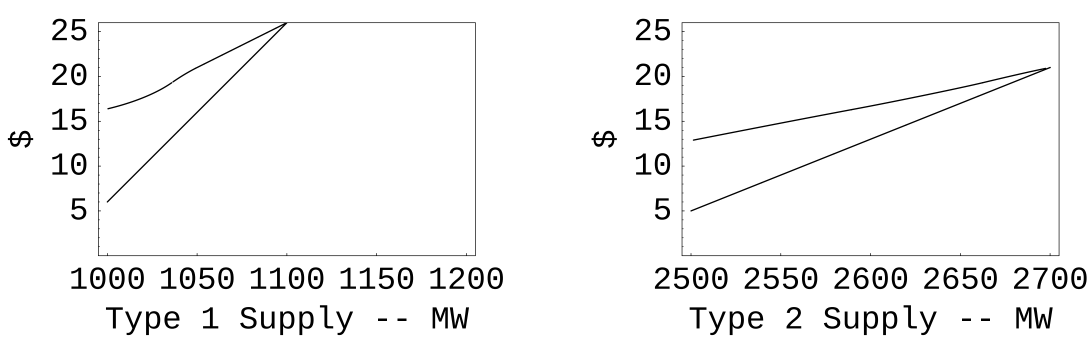

Figure 3: Bid functions for the PaB auction (Data Set 1).

One can now use the bid functions to calculate expected payments made by the ISO in a PaB auction. Figure 4 shows expected profit of equal cost generators and one can see that the PaB auction is also incentive compatible for this data set.

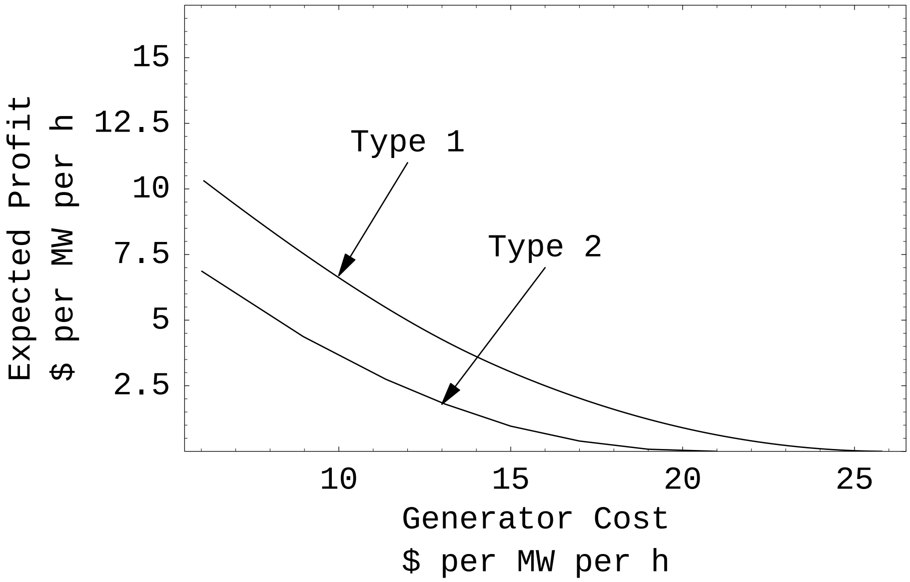

Figure 4: Expected Profit in a PaB Auction (Data Set 1).

4.4 Comparison of Expected Payments and Efficiency Properties

We can now compare expected payments across the auctions and examine their efficiency properties. Figure 5 shows expected payments for the three auction formats, while Table 1 shows the expected payments made by the ISO when conducting these auctions along with expected social cost of production.

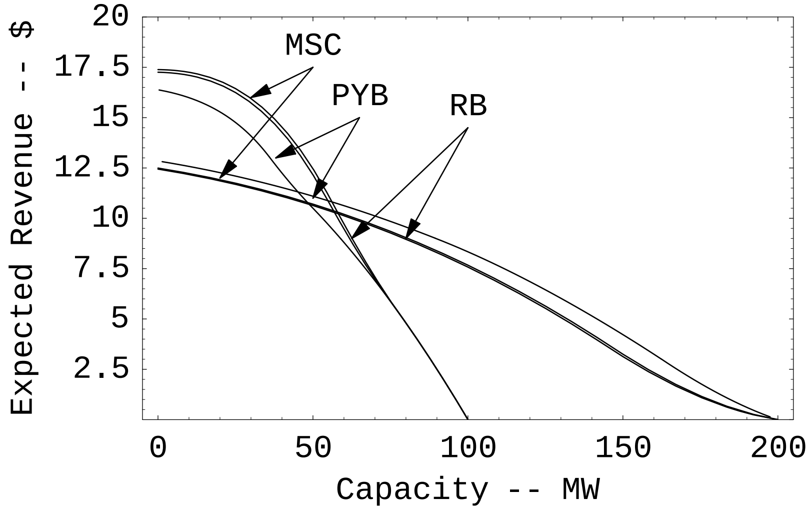

Figure 5: Comparison of Expected Payments across Auctions (Data Set 1).

Observe that for this data set the RB auction is almost equivalent to the MSC auction

	Type 1	Type 2	Total	Social Cost
MSC	18340.7	32639.4	50980.1	20143.1
RB	18485.7	32494.4	50980.1	20143.3
PaB	17376.7	33625.3	51002.0	20151.4

Table 1: Expected Payments by the ISO and Social Cost (Data Set 1)

in aggregate terms. Type 1 generators are paid a little more in expected terms in the RB auction than in the MSC auction. The reverse holds true for type 2 generators. This can be explained by comparing the prices that these generators get paid when type 1 procurement is greater than type 1 demand. Under the RB auction:

$$\begin{aligned} p_1^{case1,RB}(d_1,d_2) &= \beta_1 + \gamma_1 q_1(d_1,d_2) \\ &= \beta_1 + \gamma_1 \left(\frac{\beta_2 - \beta_1}{2(\gamma_1+\gamma_2)} + \frac{\gamma_2}{\gamma_1+\gamma_2}(d_1+d_2) \right) \end{aligned} \tag{19}$$

On the other hand, under the MSC auction:

$$\begin{aligned} p_1^{MSC}(d_1,d_2) &= \beta_3(d_1) + \gamma_3 d_2 \\ &= \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\left(\frac{\beta_1}{\gamma_1} + \frac{\beta_2}{\beta_2} + d_1 \right) + \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} d_2 \end{aligned} \tag{20}$$

The term multiplying total demand is the same in both cases. The constant term is 5.643 for the RB auction for Data Set 1, while it is 5.286 for the MSC auction. This implies that the RB auction distorts the efficient choice of procurement quantities, although by a small amount in this case.

The striking feature of the results is that in expected terms, the PaB auction pays a significantly lower sum to some type 1 generators as compared to the other two auction formats. Again, the reverse holds true for type 2 generators. A 0.04 percent increase in total expected payments is seen from the MSC auction. This is in contrast to the single demand case results (see Green and McDaniel, 1999a) where the PaB auction was shown to be equivalent to an MSC or uniform price auction. We will examine revenue equivalence in greater detail in the next section.

For type 1 generators below 75 MW in the merit order the three auctions pay the same in expected terms. This is because these generators have zero probability of selection for type 2 service and are therefore in a single-demand environment where all three auctions are equivalent. Let us examine a generator higher in the merit order, say, the generator at 50 MW in the type 1 merit order. This generator bids \$21 in the PaB auction and has a probability of selection of 0.5 (this generator is never selected for type 2 service as $q_2(21) = 200$). Thus, our generator has expected revenue of \$10.5. In the MSC auction the probability of selection is greater than 0.5 (the probability that it is selected for type 1 service is itself 0.5 and it can also be selected when demand for type 1 demand is lower than 50 MW for type 2 service). This implies that the PaB auction also distorts the efficient choice of procurement quantities.

4.5 No Expected Revenue Equivalence Between PaB and MSC Auctions

In the previous section we pointed out, that in expected terms, the PaB auction does not pay the same amount as the MSC auction to generators in the two merit orders. This implies that weak form of revenue equivalence does not hold for the hierarchichal products case. In this section we examine this in greater detail. We write the von Neumann-Morgenstern utility of a generator as simply its profit function:

$$u(c_i, P) = P - c_i \Uparrow_{c_i selected} \tag{21}$$

where c_i is its cost, $\Uparrow_{c_i selected}$ denotes the indicator for the event where the generator at cost c_i is selected for service, and P is the payment made to the generator. For a given data set, the generator can be seen as participating in a direct revelation mechanism which implements the social choice function:

$$f(.) = (\Uparrow_{c_i selected}, P) \tag{22}$$

which specifies whether a generator with cost c_i is selected and what payment it receives for all generators that can be potentially selected for service. Expected utility for the generator at c_i who reveals a cost of $\hat{c}_i$ when everyone else reveals their true cost is:

$$U(\hat{c}_i; c_i) = E[P(\hat{c}_i)] - c_i Prob\{\hat{c}_i selected\} \tag{23}$$

where the expectation is over the two demand uncertainties.

Incentive compatibility requires that generators reveal their costs truthfully in this direct revelation mechanism. Neccessary and sufficient conditions for incentive compatibility can be shown to be (see Mas-Collell et al. 1995, pp. 888-889) :

1. $Prob(c_i)$ is non-increasing.
2. $U(c_i) = \int_{c_i}^{c_i^{max}} Prob\{cselected\}$dc.

This takes into account that the mariginal generator in the stack gets selected with zero probability. We can also write this in terms of quantity variables with a change of variables, $c_i = c_i(s_i)$. The above equation means that expected utility of any generator is only a function of the probability of selection of generators lower than itself in the merit order. For two auction formats to be revenue equivalent in the weak form this probability of selection would have to be the same across the auction formats. We saw that for both, the RB and the PaB auctions, generators do not have the same probability of selection as in the MSC auction under Data Set 1. In order to see if this is possible under some other data set we write down the probability of selection for the PaB and MSC auction formats.

We can write the probability of selection of a type 1 generator at s_1 in the merit order (corresponds to cost c_1) as:

$$Prob_{MSC}(s_1 selected) = \Big((1-F_1(s_1)) + \int_0^{s_1} (1-F_2(s_1+q_2(c_1(s_1))-s))\,\mathrm{d}F_1(s)\Big) \quad (24)$$

$$Prob_{PaB}(s_1 selected) = \Big((1-F_1(s_1)) + \int_0^{s_1} \big(1-F_2\big(s_1+b_2^{-1}(b_1(s_1))-s\big)\big)\,\mathrm{d}F_1(s)\Big)$$

For these to be equal, we should have that the quantity of supply available at $b_1(s_1)$ in the type 2 bid-based merit order should be the same as that available at $c_1(s_1)$ in the original cost-based merit order, or that the bid premia for both types is the same in the PaB auction. We showed above that for Data Set 1 these auction formats will not yield the same probability of selection for a generator at 50MW in the type 1 merit order. This argument can be extended more generally by seeing that as type 1 costs dominate type 2 costs in this data set, the marginal generator for each type is at different costs, type 1 being at $q_1 = 100$ (cost $= 26$) while type 2 being at $q_2 = 200$ (cost $= 21$). This means that the type 1 generator at cost equal to 21 will have a strictly positive probability of selection while that in the type 2 merit order at 21 will have zero probability of selection. The type 1 generator will therefore have a strictly positive bid premium and the probability of selection for these generators will not be the same.

In this section we examined the RB auction under data for which it is incentive compatible. If the RB auction is incentive compatible then it performs as well as the other auction formats analyzed above. We now study some cases where the RB auction may not perform as well.

5 Price Reversals

In many electricity markets across the U.S., the occurrence of price reversals (i.e. lower type generators are paid higher prices than higher type generators) in reserves markets is cited as a major factor that is leading to extensive redesign of these markets. Proposition 1 showed that this will not occur in an MSC auction and therefore an MSC auction is always incentive compatible. However, the RB auction is susceptible to price reversals (see Oren). We examine Data Set 2 in this section and show that when type 2 costs dominate type 1 costs there are price reversals in the RB auction and this leads to a violation of incentive compatibility constraints in a cRB auction. Type 1 generators make lower expected profit than equal cost type 2 generators. Therefore, these generators will have an incentive to misrepresent their true capability. To demonstrate this incentive problem, we relax the assumption that generators have to bid their true type and simulate equilibria for the (unconstrained) RB (uRB) auction using discretized versions of the cost functions. Misrepresentation by generators leads to kinks and discontinuities in the equilibrium merit orders and analytical techniques cannot be used in this case. We find that in equilibrium a band of type 1 generators declare themselves as type 2 to secure higher expected profit. We also show that the PaB auction does not suffer from such incentive problems.

5.1 MSC Auction for Data Set 2

We can use Proposition 2 to calculate expected profit (after subtracting cost from the price terms in all the integrals) in the MSC auction for Data Set 2. These are plotted in Figure 6 which indicates that the MSC auction is incentive compatible for this data.

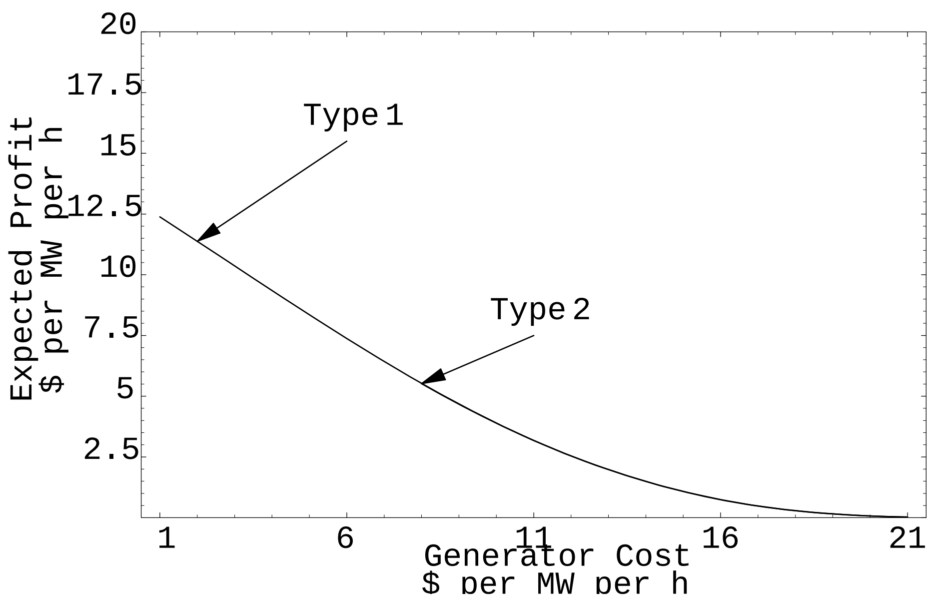

Figure 6: Expected Profit in an MSC Auction (Data Set 2).

5.2 Rational Buyer Meets Rational Seller - RB Auction for Data Set 2

We begin by examining the cRB auction. For Data Set 2, expected profit in the cRB auction cannot be calculated using Proposition 5 as conditions for the analytical solution to be valid are violated. Instead, we discretize the problem and solve the optimization problem using the dynamic programming algorithm outlined in Section 4.2 for each pair of demand realizations. Figure 7 shows expected profit for the cRB auction.

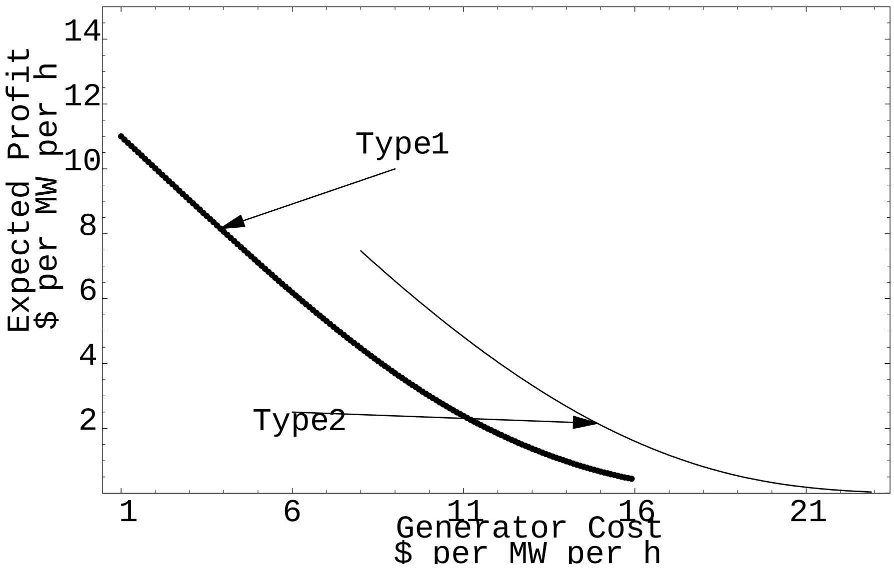

Figure 7: Expected Profit in a cRB Auction (Data Set 2).

Observe that unlike Data Set 1, type 1 generators get lower expected profit than equal cost type 2 generators under this data. This implies that if a type 1 generator unilaterally misrepresents its type, it would make higher expected profit as a type 2 generator. There may be more than one generator who may wish to misrepresent their capability. This would lead to discontinuities in the type 1 merit order. Depending on the costs of the generators that misrepresent their type there would be discontinuities or kinks in the type 2 merit order as well. To simulate an equilibrium we use an iterative algorithm which moves type 1 generator over to the type 2 merit order and then calculates expected profit using the dynamic programming solution for each demand realization iterating until no generator finds it profitable to move. The algorithm is implemented in the C programming language. Figure 8 shows the equilibrium cost functions for the uRB auction. Type 1 generators with costs between \$1.9 and \$5.0 (32MW) pretend to be type 2. Figure 9 shows expected profit of equal cost generators in the two merit orders. The thick curve plots expected profit of type 1 generators (note that a band of generators are missing from this merit order). The thin curve in the gap are type 1 generators that have misrepresented their capability and other equal cost type 2 generators.

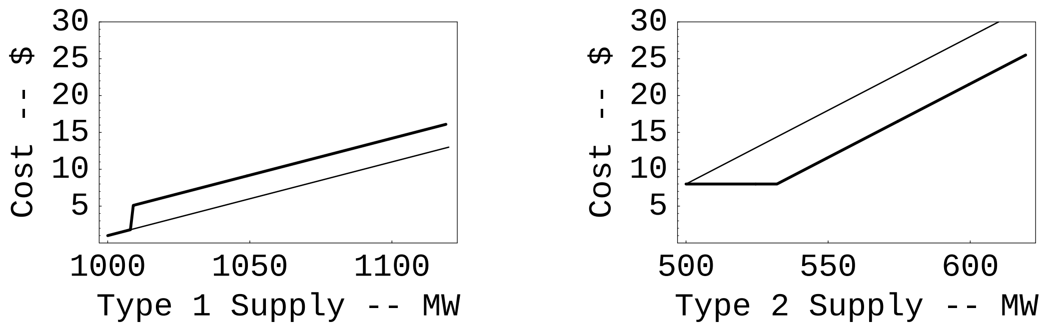

Figure 8: Equilibrium Cost Functions in an uRB Auction (Data Set 2).

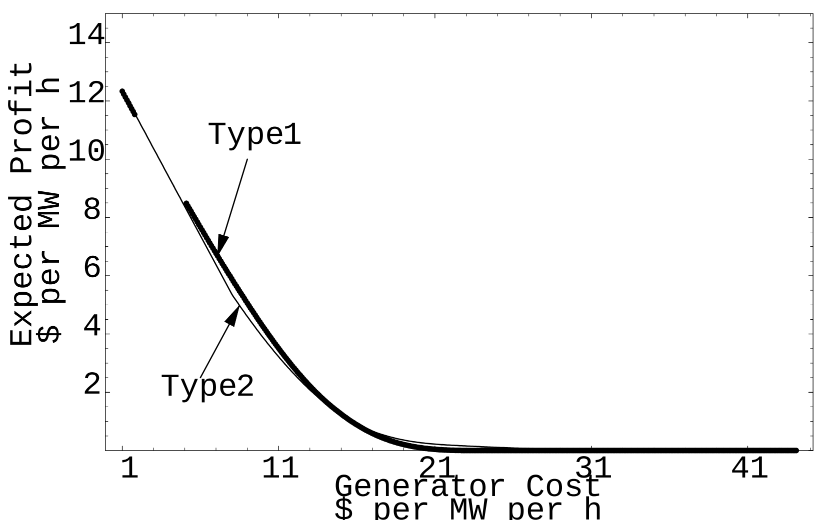

Figure 9: Expected Profit in an unconstrained RB Auction (Data Set 2).

We also plot the price duration curve for the equilibrium (see Figure 10). This plot has percentages of time or duration on the horizontal axis and plots the minimum price seen for that duration or less.

5.3 PaB Auction for Data Set 2

We use a similar approach as in Section 4.3 to calculate bid functions in the PaB auction. We first determine generators that will be selected with zero probability in each merit order (This time there will be no Part 1 of the analysis because type 2 costs dominate type 1 costs considering only the uncertain parts of the demand) and then solve the differential equations in several steps as before (see Figure 11 for a plot of the bid functions). We find

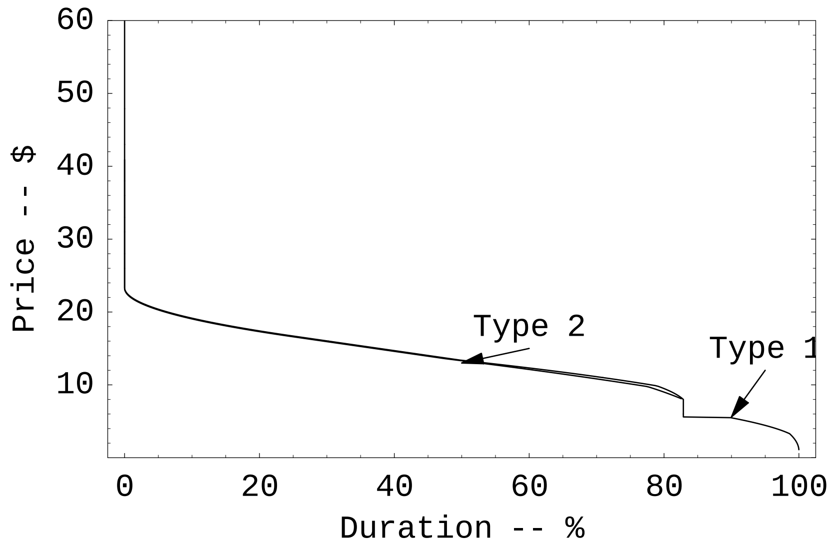

Figure 10: Price Duration Curve in an RB Auction (Data Set 2).

that type 1 generators make marginally higher profit than equal cost type 2 generators and this auction is incentive compatible (see Figure 12).

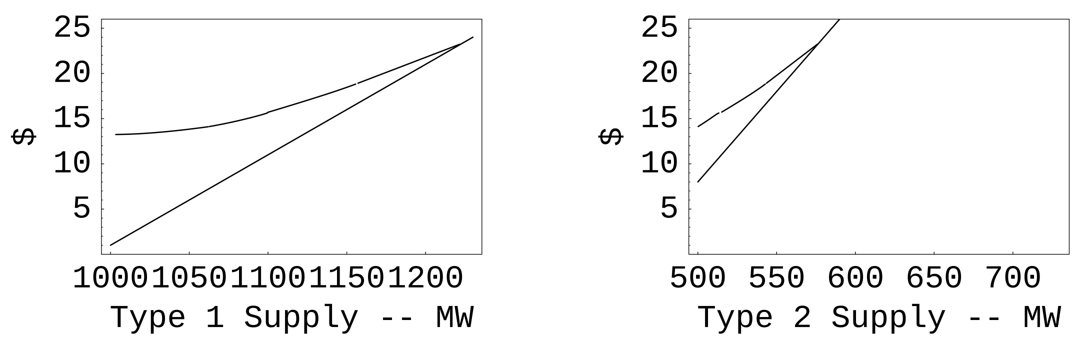

Figure 11: Bid Functions in a PaB Auction (Data Set 2).

5.4 Comparison of Expected Payments and Efficiency Properties

In this section we again address the question of how expected payments compare across the auction formats. We showed above that the MSC and PaB auctions are incentive compatible, while in the RB auction there will be some misrepresentation of capability by higher type generators.

Again, the PaB auction results in higher payments than the MSC and RB auctions for

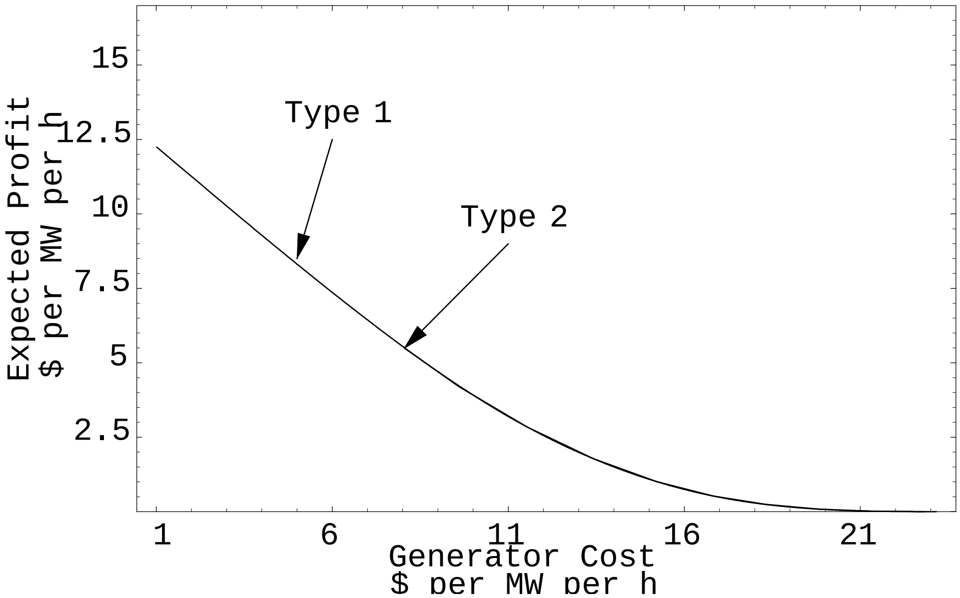

Figure 12: Expected Profit in a PaB Auction (Data Set 2).

	Type 1	Type 2	Total	Social Cost
MSC	15382.1	6721.4	22103.5	5813.1
cRB	13521.9	8369.8	21891.7	6341.1
uRB	14830.8	7592.0	22422.8	6252.2
PaB	15988.8	6826.1	22814.9	6308.4

Table 2: Expected Payments by the ISO and Social Cost (Data Set 2)

this data. The uRB auction results in about 0.9 percent higher expected payments than the MSC auction in this case. Misrepresentation of capability by generators leads to an increase of 1.8 percent in expected payments wiping out any potential savings that the cRB auction had over the MSC auction.

5.5 No Expected Revenue Equivalence under Data Set 2

We saw in Section 4.5 that bid premia for generators in the two merit orders will not be the same and therefore we do not get weak form revenue equivalence for Data Set 1. This result was based on the fact that marginal generators in the two merit orders do not have equal cost and therefore equal cost generators will have different probabilities of selection and therefore unequal bid premia. For Data Set 2, however, marginal generators have equal cost. One way to check if revenue equivalence holds in this case is to use the MSC probability to calculate the bid function analytically and check first order conditions for the maximization problem in 10. This approach may run into difficulty as the first order conditions call for inverting the type 2 bid function which may be difficult. A heuristic argument can still be made that the probability of selection of a type 1 generator bidding cost in a PaB auction dominates that of an equal cost type 2 generator. As generators trade off bidding above marginal cost against lower probability of selection, this implies that they will not have the same bid premia and thus weak form revenue equivalence does not go through again.

6 Capacity Constraints

In the previous sections we examine the three auction formats under the assumption that there are no capacity constraints on any service type. In this section, we examine reserves auctions with capacity constraints. We feel it is important to examine how the market will perform under stress as misrepresentation in these times could have an impact on reliability. We take two data sets, Data Set 3 and Data Set 4,, such that type 1 costs dominate type 2 costs (considering the entire merit order including certain demand portions). Data Set 3 assumes that total type 1 capacity is only 5 MW more than the upper support of type 1 demand, i.e. it is 1105 MW. In Data Set 4, type 1 capacity is as above with type 2 capacity 10MW short of the upper support of type 2 demand, i.e. it is 2690 MW. We assume that prices will be set to pre-specified price caps in demand states where there are shortages. We analyze results for three price caps of \$100, \$250 and \$500, which we assume are equal for both types of resources. We show that such shortages indeed occur in an RB auction by simulating approximate equilibria for the three price cap levels. For Data Set 3, we find that the frequency of price spike reduces as the price cap level is increased. For Data Set 4, the frequency of price spikes reduces for type 1 resources while increasing for type 2 resources when the price cap level is increased. We also show that the PaB and MSC auction formats are not susceptible to type 1 shortages if there is non-negative surplus of type 1 capacity and that the level of shortage for type 2 resources will not be sensitive to the price cap level.

6.1 MSC Auction under Data Set 3

The MSC auction is incentive compatible for Data Set 3 (see Figure 13). An immediate consequence of this is that there will be no type 1 shortages in an MSC auction under type 1 capacity constraints as long as there is sufficient capacity to cover the range to type 1 demand.

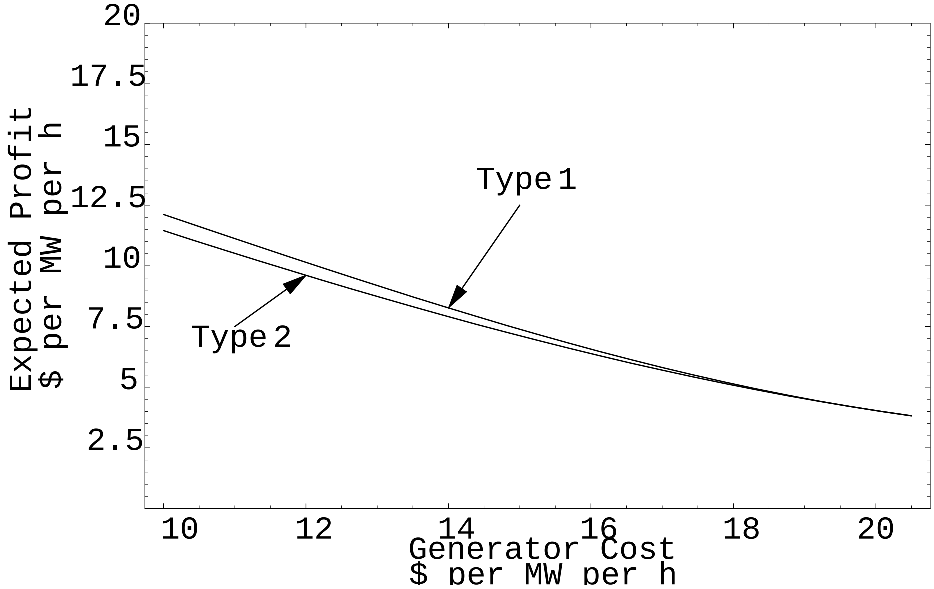

Figure 13: Expected Profit in a MSC Auction with Type 1 Capacity Constraints (Data Set 3).

6.2 Rational Buyer under Data Set 3

Rational bidders in a hierarchical products auction where price reversal takes place will want to misrepresent their capability to secure higher profit. If there are capacity constraints on type 1 resources this could lead to artificial shortages in type 1 capacity. Figure 14 shows expected profit in a cRB auction with capacity constraints on type 1 capacity. Type 1 generators make much lower expected profit than equal cost type 2 generators in this case and will have incentive to misrepresent capability.

Figure 16 shows equilibrium expected profit for a price cap of $100 in a uRB auction (see Figure 15 for equilibrium cost functions). We find that in equilibrium, the level of shortages will depend on the price cap. When the price cap is $100, shortages occur about 5.5 percent of the time (see Figures 17 and 18 for price duration curves). Increasing the price cap to $500 reduces this to 0.5 percent of the time. This is because the premium paid to generators is larger for higher price caps and more type 1 generators have incentive to reveal their true capability in order to take advantage of the price spike periods.

6.3 Pay as Bid Auction under Data Set 3

The PaB auction does not suffer from incentive compatibility problems like the RB auction above under capacity constraints. Thus, if there is sufficient type 1 capacity to begin with, we will not see type 1 shortages in this auction. We approach the equilibrium bid

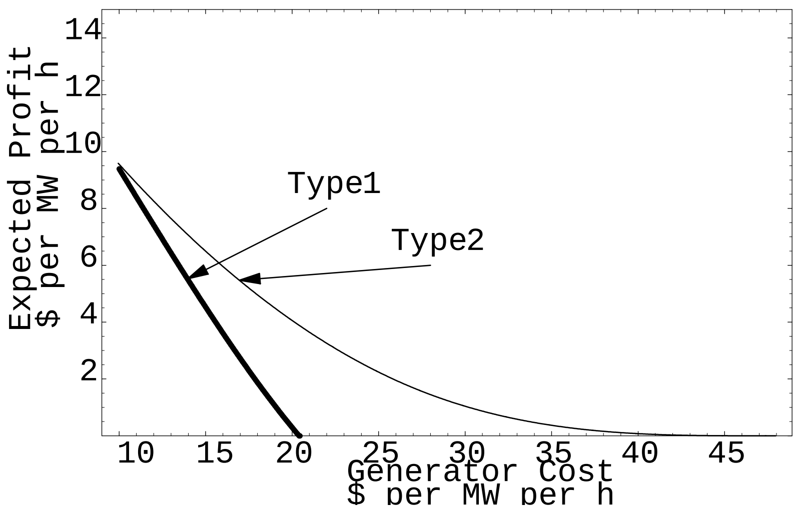

Figure 14: Expected Profit in a cRB Auction with Type 1 Capacity Constraints (Data Set 3).

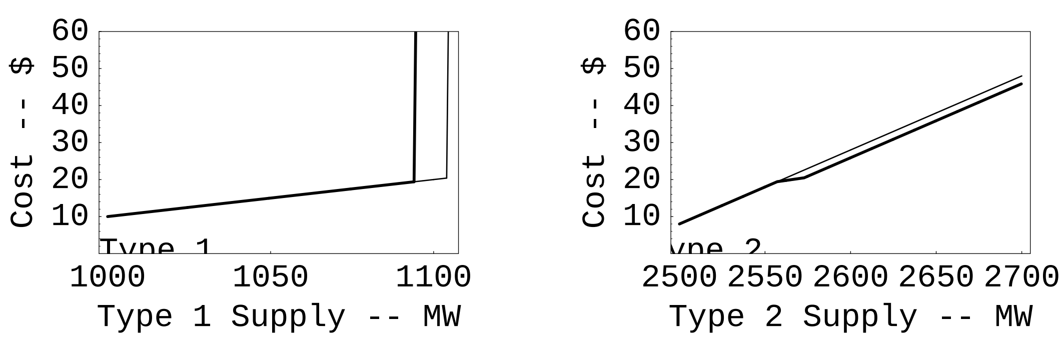

Figure 15: Equilibrium Cost Functions in a uRB Auction with Type 1 Capacity Constraints (Data Set 3; price cap = \$100).

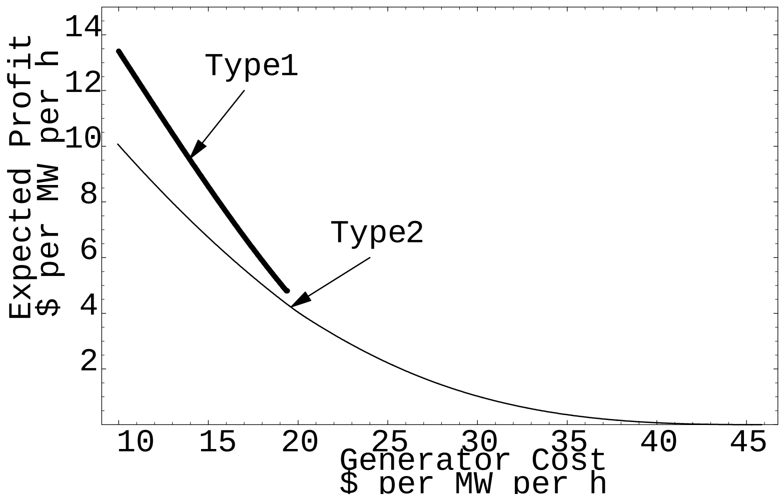

Figure 16: Expected Profit in a uRB Auction with Type 1 Capacity Constraints (Data Set 3; price cap = $100).

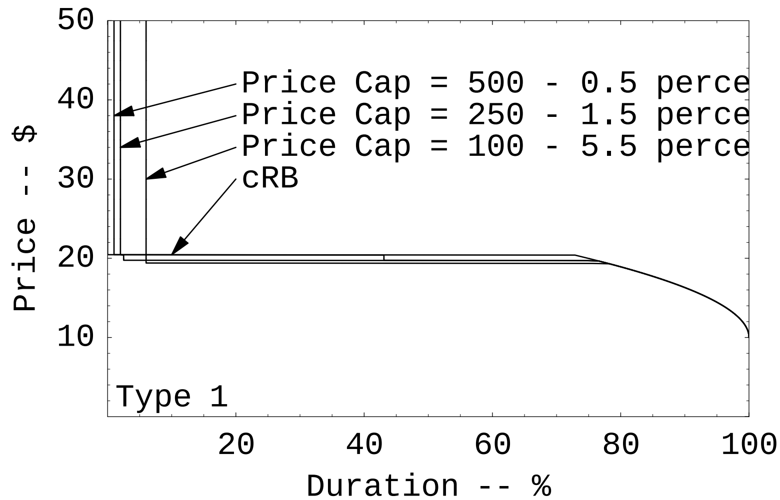

Figure 17: Price Duration Curves in a uRB Auction with Type 1 Capacity Constraints (Data Set 3; Type 1).

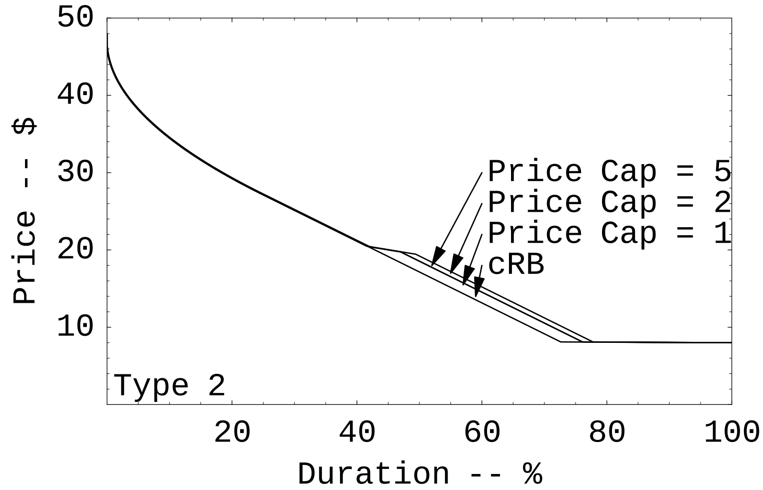

Figure 18: Price Duration Curves in a uRB Auction with Type 1 Capacity Constraints (Data Set 3; Type 2).

calculations in the same manner as before. The highest cost type 1 generator will be able to capture some rent in market because it is chosen with strictly positive probability (see Figure 19 for bid functions and Figure 20 for expected profit under this auction format).

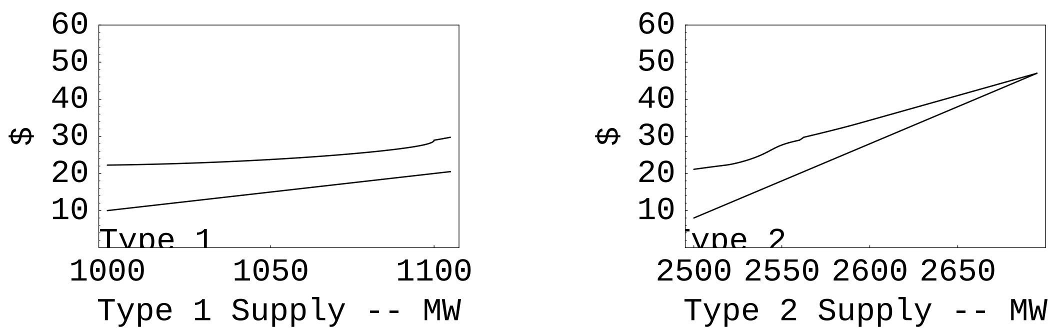

Figure 19: Bid Functions in a PaB Auction with Type 1 Capacity Constraints (Data Set 3).

6.4 Comparison of Expected Payments and Efficiency Properties

Table 3 compares expected payments by the ISO for Data Set 3 with capacity constraints on type 1 capacity. The ranking of the PaB and MSC auctions seems to have reversed for Data Set 3 as compared to the previous data sets where the PaB had higher expected payments by the ISO. This result echoes the ambiguous ranking results for these auctions

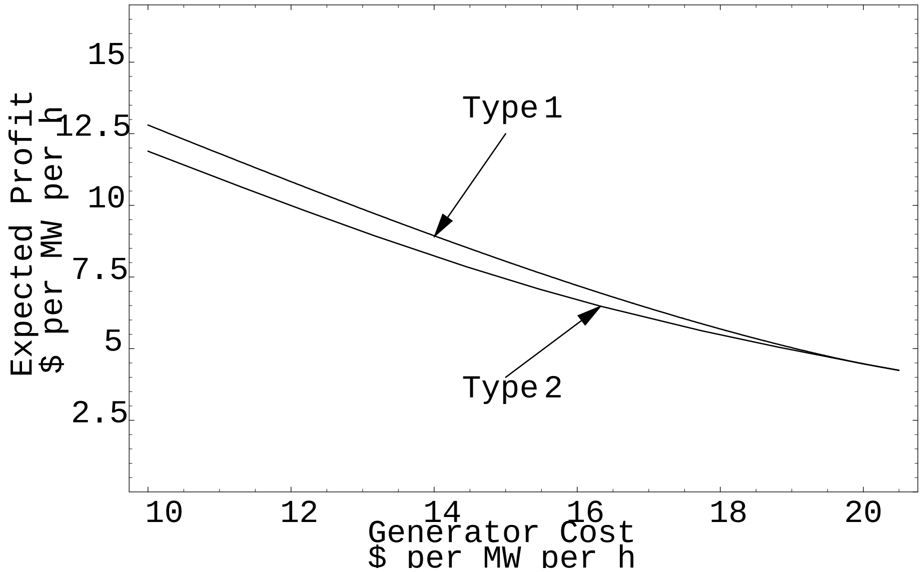

Figure 20: Expected Profit in a PaB Auction with Type 1 Capacity Constraints (Data Set 3).

in the literature (e.g. see Ausubel and Cramton). The cRB auction in this case has 11.0 percent lower expected payments than the MSC auction. This is due to the fact that because of capacity consraints the MSC auction pays a premium over marginal cost to type 1 generators under many demand realizations states, the RB auction can avoid such premiums by procuring just under the capacity limit. Now, though the two auctions have the same payment rule of paying marginal cost of providing a reserve type, the RB auction pays marginal cost of the last selected generator under this rule. More than half of this difference is made up in the unconstrained RB auction and the ISO has to pay about 3.8 percent lower expected payments for the $250 price cap than the MSC auction. Prices, however, will be a lot more volatile in the RB auction staying at the price cap level 0.5-5.5 percent of the time.

The last column in the table shows the expected social cost for the different auction formats. It can be seen that the RB auction trades off efficiency for lower expected payments. In order to accurately account for social cost incurred during times of shortages we use a value of lost reliability in our social cost calculations whenever demand exceeds type 1 capacity. We assume this value to be $1500 per MW of shortage. Explicitly including the cost of shortage in the social cost calculations produces the result that as the price cap increases the social cost of the RB auction decreases as shortages occur less frequently. The PaB auction for Data Set 3 is very close to the MSC auction in terms of social cost. For this data the bid premia of the two types are almost equal. The Type 1 bid premuim is

	Type 1	Type 2	Total	Social Cost
MSC	24109.8	55080.9	79190.7	32373.6
cRB	21338.0	49162.1	70500.1	32438.9
uRB-100	25606.3	50826.5	76432.8	32663.5
uRB-250	25721.9	50439.5	76161.4	32450.1
uRB-500	26275.8	50321.5	76597.3	32430.0
PaB	24285.4	54564.5	78849.9	32374.0

Table 3: Expected Payments by the ISO and Social Cost (Data Set 3).

greater for low cost type 1 generators than equal cost type 2 generators. However, lower down in the merit order it is lower than the bid premium for an equal cost type 2 generator. Ineffiencies are small in any case and the auction is almost efficient.

6.5 Data Set 4

In this section we examine the case where there are capacity constraints on both type of resources. We assume the same data as Data Set 3 above with the additional constraint that type 2 capacity is 10 MW below the upper support of type 2 demand. This implies that there will be some type 2 price spikes in the auctions. We find that the MSC and PaB auctions continue to be incentive compatible (figures are not reported here as they look very similar to the ones for Data Set 3). There are not shortages for type 1 capacity and the amount of shortage for type 2 capacity is not sensitive to the price cap that is paid to capacity in periods of shortage. One difference between the RB auctions and the above two is that the level of shortage for type 2 resources is sensitive to the price cap in an RB auction. We illustrate this point below. The frequency of price spikes in a cRB auction is about 0.14 percent, much smaller than the 5 percent shortage that we assume for type 2 capacity. This is because most of the time there is enough surplus of type 1 capacity that the ISO can procures to avoid any shortage. Due to the nature of the rational buyer protocol, however, the ISO will never pay type 1 capacity at the price cap level as it will always procure an amount a little lower than the type 1 capacity level. Figure 21 shows expected profits in a cRB auction for this case. Type 1 generators have a similar incentive as the previous case as they make much smaller expected profits than equal cost type 2 generators.

Figure 22 shows the equilibrium cost functions for a price cap level of $250 (See Figure 23 for equilibrium expected profits). In equilibrium there is a 2.0 percent shortage of type 1 capacity while there is a 0.12 percent shortage in type 2 capacity. This is reflected in the

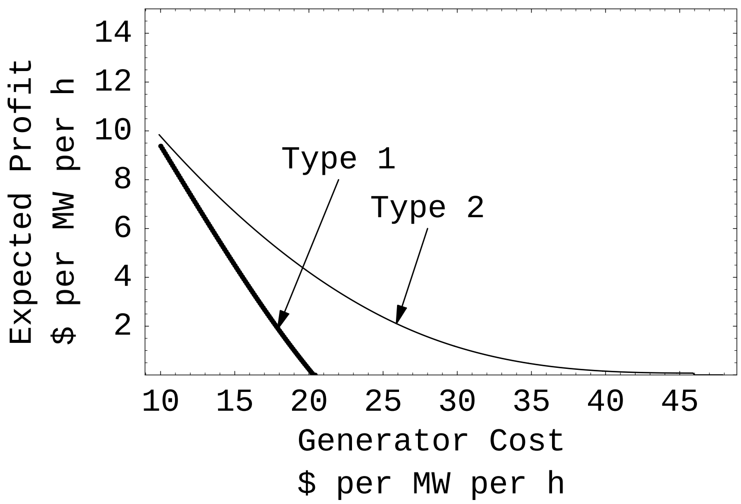

Figure 21: Expected Profit in a cRB Auction with Capacity Constraints in both Types (Data Set 3).

price duration curves in Figure 24 and 25, where it can be seen that type 1 prices stay at the price cap level about 2.0 percent of the time for a price cap of \$250, while the frequency of price spikes in the type 2 case is reduced. Increasing the price cap level has the effect that it decreases the frequency of price spikes for type 1 but increases the frequency for type 2 as more resources prefer to reveal their true capability.

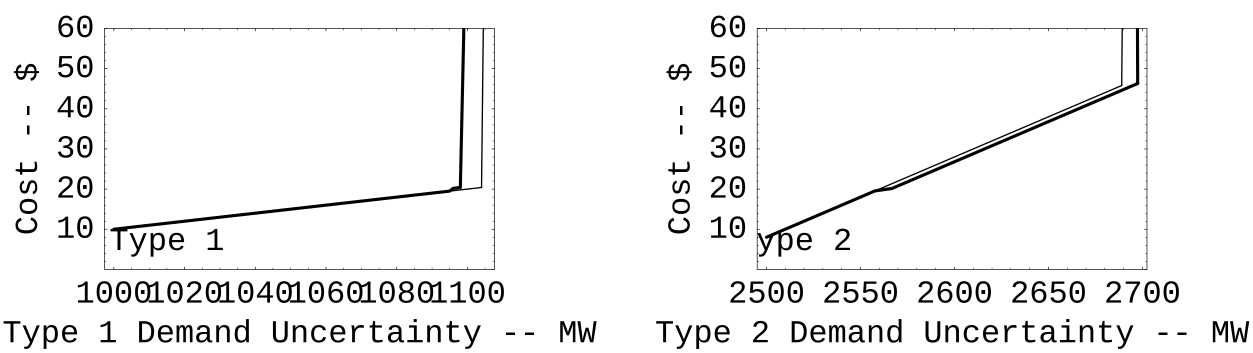

Figure 22: Equilibrium Cost Functions in a uRB Auction with Capacity Constraints in both Types (Data Set 4; price cap = \$250).

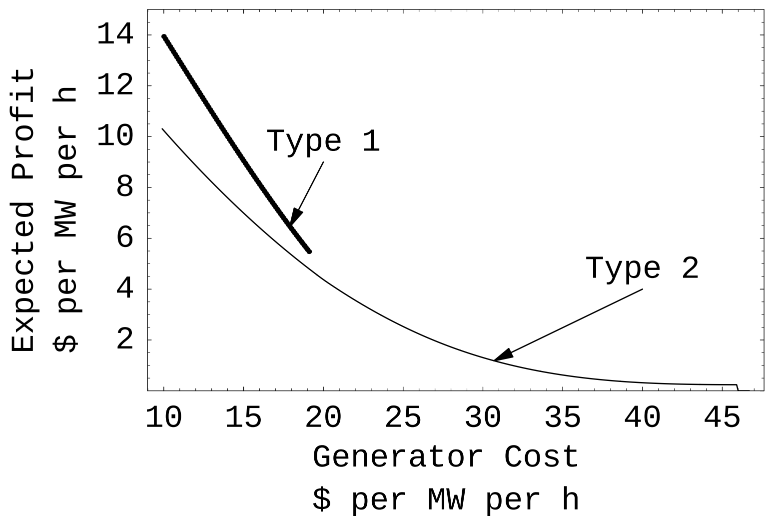

Figure 23: Expected Profit in a uRB Auction with Capacity Constraints in Both Types (Data Set 4; price cap = $250).

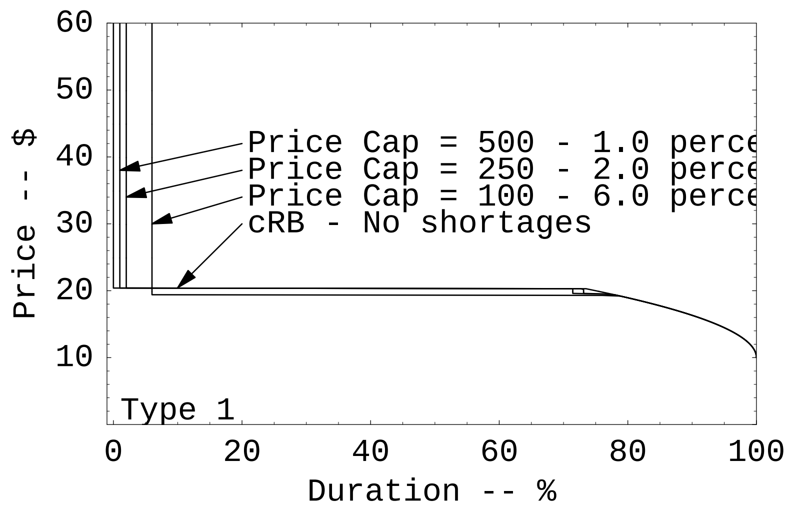

Figure 24: Price Duration Curves in a uRB Auction with Capacity Constraints in Both Types (Data Set 4; Type 1).

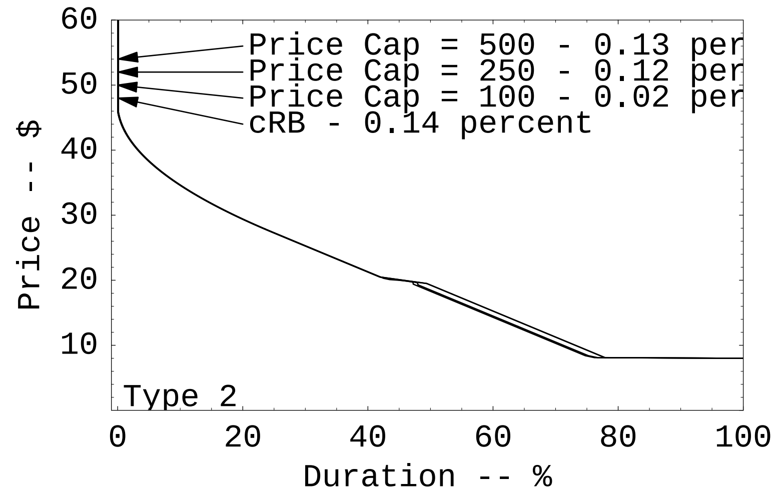

Figure 25: Price Duration Curves in a uRB Auction with Capacity Constraints in Both Types (Data Set 4; Type 2).

	Type 1	Type 2	Total	Social Cost
MSC-250	24251.2	55423.3	79674.5	32375.8
cRB	21251.5	49605.5	70857.0	32445.3
uRB-250	26172.7	51057.1	77229.8	32448.9
PaB-250	24285.5	54564.4	78849.9	32376.2

Table 4: Expected Payments by the ISO and Social Cost (Data Set 4).

6.6 Comparison of Expected Payments and Efficiency Properties for Data Set 4

Table 4 compares expected payments and social costs across the three auction formats for a price cap level of $250.

7 Concluding Remarks

We analyze three auction formats for a two-product ancillary services market that provides reliability products in the real time operation of an electricity network. Using a stylized model and various numerical examples we study efficiency properties and incentive compatibility of a uniform price auction minimizing social cost, a uniform price auction minimizing the ISO's cost and a pay as bid auction. We show that under the assumptions of our model, the MSC auction is the only one that guarantees productive efficiency. We also examine expected revenue equivalence between pay as bid and uniform price auctions and find that standard results do not extend to the hierarchical products case. For the minimum procurement cost auction that has been adopted under the "Rational Buyer" protocol in California we show that though this auction is incentive compatible for some data, when there are capacity constraints on high quality resources, misrepresentation of capability by generators may lead to shortages of capacity for high quality resources and random spikes in prices. For the case where only higher capability resources are constrained this will result in random price spikes decreasing in frequency with the price cap which is paid to all available capacity in shortage periods. When lower type resources are capacity constrained as well, the shortage will spread to the higher type resources and price spikes will be seen for both type of resources in equilibrium.

We find that the Pay as Bid auction is not susceptible such incentive compatibility problems and performs well under conditions of stress on the system. Creating separate products differentiated by readiness, flexibility and location, fragments markets and clearing each market at uniform prices created thin markets with potential market power problems. Commoditization of electricity seen in forward markets is greatly reduced in real-time markets as each resource is capable of providing a unique service in real-time operations. A PaB auction allows such resources to internalize their differences by paying each resource its bid. This also eliminates the price reversal problem and neutralizes the "last man in" strategy seen in uniform price markets with generators that have market power. A drawback of the PaB auction, however, is that the discriminatory nature of the auction makes it susceptible to bypass.

Our stylized model uses infinitesimal generators in a competitive market. In a working electricity market however, one deals with generators having market power. We hope to evaluate the performance of these auction formats under this assumption in future research.

Appendix

Parameter	Data Set 1	Data Set 2	Data Set 3	Data Set 4
β_1	6	1	10	10
γ_1	0.20	0.10	0.10	0.10
β_2	5	8	8	8
γ_2	0.08	0.20	0.20	0.20
r_{10}	1000	1000	1000	1000
r_{20}	2500	500	2500	2500
r_1	100	100	100	100
r_2	200	200	200	200
Type 1 Constrained	No	No	Yes	Yes
Type 2 Constrained	No	No	No	Yes

Table 5: Data Used in the Numerical Examples

Proof of Proposition 1: Let $\lambda_i, i = 1, 2, \dots n$ denote the Lagrange multipliers on the constraints. For general weakly increasing supply functions (continuity is not required; any ties that occur can be broken by prorating all equal cost supply) we can write the price paid to each generator for a particular demand type i as:

$$p_i = \sum_{j=i}^{n} \lambda_j \tag{25}$$

As all the Lagrange multipliers are positive, this implies that $p_1 \geq p_2 \dots \geq p_n$ (see Oren 2000 for an example where the problem can be solved as a Linear Program).

Proof of Proposition 2: A type 1 generator at s_1 in the merit order will be selected for type 1 service if type 1 demand is greater than s_1. For a demand of d_1 ($> s_1$), the price that this generator will be paid will depend on whether any type 1 generators that spilled over to the type 2 auction were selected for service. There will be a type 2 demand level, $d_2^* = q_2(c_1(d_1))$, such that if type 2 demand is less than d_2^*, no type 1 generator is selected for type 2 service. In this case both types will be paid different prices based on the cost of the marginal generators in each merit order. At demand levels greater than d_2^* (if they exist), some type 1 generators will be selected for type 2 service and both types will be paid the same price. Finally, when type 1 demand is below s_1 our generator will be selected whenever type 2 demand is greater than $s_1 - d_1 + q_2(\beta_1 + \gamma_1 s_1)$ (if it is within its range). It will be paid the price of the marginal generator selected for type 2 service. Substituting the appropriate limits and prices when taking expected values gives us the formula in the proposition.

For type 2 generators there will also be three cases. In the first case, type 1 demand is greater than the quantity corresponding to this generators cost, $q_1(c_2(s_2))$ and type 2 demand is such that no type 1 generator is selected for type 2 service. In the second case, type 1 demand is as above but there are some type 1 generators that are selected for type 2 service and both types are paid the same price. In the third case, type 1 demand is less than $q_1(c_2(s_2))$ and when this generator is selected some type 1 generators are also selected and both are paid the same price. One can substitute these limits and prices in the expected revenue formula and arrive at the formula in the proposition.

Proof of Proposition 3: We first examine the best response of an arbitrary type 1 generator in an constrained RB auction when everyone else bids cost. Now, an infinitesimal type 1 generator at s_1 in the merit order will be selected for service whenever demand for type 1 service is $> s_1$. In this case, altering its bid does not change the prices it receives, while it only reduces the probability of selection. When type 1 demand is $< s_1$, there is still a positive probability that this generator is selected for service. This will happen for demand realizations where Case 1 applies (When type 1 demand is sufficiently low and type 2 demand sufficiently high to justify procuring more type 1 service than demanded). In this case, too, our generator cannot change prices it receives, while bidding above cost reduces the probability of selection. For demand realizations that this generator will not be selected, prices are below its cost and it rather not be selected for service. Bidding below cost will increase the probability of selection of our generator but all the added demand realizations will be such that type 1 prices are below its cost. One can follow this reasoning for a type 2 generator as well. Therefore, bidding cost results in an equilibrium in the constrained RB auction.

Proof of Proposition 4: Let us examine the optimal procurement quantities under the two cases in the KKT conditions. For demand realizations that fall under Case 1 we should have:

$$c_1(q_1(d_1, d_2)) \geq c_2(q_2(d_1, d_2)) \tag{26}$$

Substituting from 7 we get:

$$\begin{aligned}&\beta_1 + \gamma_1\Big(\frac{\beta_2 - \beta_1}{2(\gamma_1 + \gamma_2)} - \frac{\gamma_1 r_{10} - \gamma_2 r_{20}}{2(\gamma_1 + \gamma_2)} + \frac{\gamma_2(d_1 + d_2)}{\gamma_1 + \gamma_2}\Big)\\ &\geq \beta_2 + \gamma_2\Big(-\frac{\beta_2 - \beta_1}{2(\gamma_1 + \gamma_2)} + \frac{\gamma_1 r_{10} - \gamma_2 r_{20}}{2(\gamma_1 + \gamma_2)} + \frac{\gamma_1(d_1 + d_2)}{\gamma_1 + \gamma_2}\Big)\end{aligned} \tag{27}$$

Dividing by $\gamma_1 r_{10}$ (and assuming that terms involving the intercepts are negligible after this division) this can be simplified to get the desired inequality.

We also need to check if price reversals can occur under Case 2. We can write an inequality for the set of demand realizations under Case 2. As procured quantities are affine functions of the two demands the two regions (for Case 1 and Case 2) will be separated by a straight line at:

$$q_1(d_1, d_2) = \frac{\beta_2 - \beta_1}{2(\gamma_1 + \gamma_2)} + \frac{\gamma_2(d_1 + d_2)}{\gamma_1 + \gamma_2} = d_1 \tag{28}$$

The inequality characterizing demand realizations under Case 2 can be written as:

$$d_2 \leq \frac{\beta_1 - \beta_2}{2\gamma_2} + \frac{\gamma_1}{\gamma_2} d_1 \tag{29}$$

No price reversals implies that for demand realizations falling under Case 2:

$$c_1(d_1) \geq c_2(d_2) \tag{30}$$

$$\begin{aligned} \beta_1 + \gamma_1 d_1 &\geq \beta_2 + \gamma_2 d_2 \\ d_2 &\leq \frac{\beta_1 - \beta_2}{\gamma_2} + \frac{\gamma_1}{\gamma_2} d_1 \end{aligned} \tag{31}$$

Now, observe that the set where no price reversals occur under Case 2 covers the set of demand realizations under Case 2.

Proof of Proposition 5: As the ISO minimizes its procurement cost, when type 1 demand is low and type 2 demand is high, the ISO can reduce cost by selecting more type 1 generators than type 1 demand (This is Case 1 above). In states where type 1 demand is high, such substitution may not reduce procurement cost and the optimal procurement involves meeting each demand with generation of that type (This is Case 2 above). As procured quantities are a linear function of the two demands the two regions will be separated by the straight line described in equation 28:

$$q_1(d_1, d_2) = \frac{\beta_2 - \beta_1}{2(\gamma_1 + \gamma_2)} + \frac{\gamma_2(d_1 + d_2)}{\gamma_1 + \gamma_2} = d_1 \tag{32}$$

The above equation can be written as:

$$d_2^*(d_1) = \frac{\beta_1 - \beta_2}{2\gamma_2} + \frac{\gamma_1}{\gamma_2} d_1 \tag{33}$$

Another boundary can be written when quantity of type 1 demand is lower than s_1 and Case 1 holds to make it equal to s_1.

$$q_1(d_1, d_2) = \frac{\beta_2 - \beta_1}{2(\gamma_1 + \gamma_2)} + \frac{\gamma_2(d_1 + d_2)}{\gamma_1 + \gamma_2} = s_1 \tag{34}$$

This can be written as:

$$d_2^{**}(d_1) = \frac{\beta_1 - \beta_2}{2\gamma_2} + \frac{\gamma_1 + \gamma_2}{\gamma_2} s_1 - d_1 \tag{35}$$

The formulae given can be derived by simply substitution appropriate limits according to the areas outlined above and using the appropriate price. $d_2^* = d_2^*(0) = \frac{\beta_1 - \beta_2}{2\gamma_2}$. d_2^{***} can be derived by solving $d_2^*(d_1) = s_1$. $d_1^*(d_2)$ is the inverse of $d_2^*(d_1)$. d_1^{**} is be derived by solving $d_2^*(d_1) = d_2^{**}(d_1)$.

Proof of Proposition 6: This is a direct consequence of the fact that there are no price reversals when $\gamma_2 r_{20} - \gamma_1 r_{10} = 0$. (see Proposition 4). There can be no state where some type 1 generator is not selected for service and an equal cost type 2 generator is selected for service (as this would imply a price reversal). In all states where both are selected for service the type 1 generator is paid at price that is no smaller than the type 2 generator. There are also states where the type 1 generator is selected for service and the type 2 generator is not. Therefore expected profit of the type 1 generator will be greater than that of the type 2 generator and the RB auction will be incentive compatible.

Proof of Proposition 7: A generator with cost $c(q_1)$ will solve:

$$\begin{aligned} \max_b \ \Pi_1(b) &= (b - c(q_1))\,(1 - F_1(s_1(b)) \\ \Rightarrow \quad & (b - c(q_1))\left(1 - f_1(s_1(b))s_1'(b)\right) + (1 - F_1(s_1(b))) = 0 \end{aligned} \tag{36}$$

This can be solved in inverse form by substituting $s_1'(b) = \frac{1}{b'(s_1)}$. This gives:

$$\frac{\mathrm{d}b}{\mathrm{d}s_1} = (b(s_1) - (\beta_1 + \gamma_1 s_1))\left(\frac{f_1(s_1)}{1 - F_1(s_1)}\right) \tag{37}$$

We can use an integrating factor, $e^{\int \mathrm{d}\log(1-F_1(s_1))}$ to get the formula in Proposition 7.

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